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**A COMBINED INVENTORY AND
DELIVERY MODEL FOR REPAIRABLE ITEMS**

THESIS

Bahtiyar Eren, 1Lt, TAAF

AFIT/GOR/ENS/00M-11

**DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY**

Wright-Patterson Air Force Base, Ohio

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ABSTRACT (<i>Maximum 200 Words</i>) Traditionally, logistics analysts have treated the delivery problem separate from the inventory and repair problems. In other words, each of these three problems—delivery, inventory, and repair—are treated individually. We combine the vehicle routing problems and inventory allocation problems in a single formulation. Furthermore, lateral re-supply is considered, in that supplies are not only available from the main depot, but also from other bases. After we get the lateral re-supply information, we set a delivery allocation plan out of the main depot. Such allocations are then delivered via vehicles stationed at the main depot. There is a repair capability for each base other than the main depot as well. There is a newsboy inventory-cost function associated with each base to the every other base. We set up a model for repairable items only. Each base has two options: accept the repairable items, or deliver the repairable items, but not both. If the base chooses the "accept" option, we make sure that that base has used all its available resources, initial inventoried items and repair capabilities, before it receives re-supplies. In the transportation/delivery submodel, we place limitation on the crew duty hours available. The objective function is to satisfy the demands by minimizing traveling cost and inventory cost. We solve the same problem by using a generalized Benders decomposition technique. The decomposition allows us to attack larger problems and use general failure and repair functions familiar to logistics analysts. Computational experience suggests that efficiency is achieved by combining the delivery and inventory functions. Lateral re-supply and computational efficiency is particularly useful in emergency situations.				
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A COMBINED INVENTORY AND DELIVERY MODEL FOR
REPAIRABLE ITEMS

THESIS

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Department of Systems and Engineering Management
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Air Force Institute of Technology
Air University
Air Education and Training Command
In Partial Fulfillment of the Requirements for the
Degree of Master of Science in Engineering and Environmental Management

Bahtiyar EREN B.S.

1st Lt., TUAF

March 2000

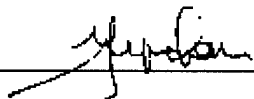
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A COMBINED INVENTORY AND DELIVERY
MODEL FOR REPAIRABLE ITEMS

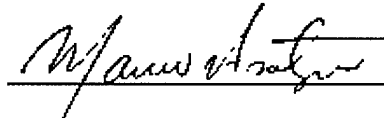
Bahtiyar EREN, B.S.
1Lt, TAAF

Approved:



Yupo Chan, Ph.D, P.E.(Chairman)
Professor of Operational Research
Department of Operational Sciences
Air Force Institute of Technology

9 Mar 00



Marvin Arostegui (Member)
Assistant Professor of Logistics Management
Department of Operational Sciences
Air Force Institute of Technology

9 Mar 00

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Bahtiyar Eren

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Abstract

Traditionally, logistics analysts have treated the delivery problem separate from the inventory and repair problems. In other words, each of these three problems—delivery, inventory, and repair—are treated individually. We combine the vehicle routing problems and inventory allocation problems in a single formulation. Furthermore, lateral re-supply is considered, in that supplies are not only available from the main depot, but also from other bases. After we get the lateral re-supply information, we set a delivery allocation plan out of the main depot. Such allocations are then delivered via vehicles stationed at the main depot.

There is a repair capability for each base other than the main depot as well. There is a newsboy inventory-cost function associated with each base to the every other base. We set up a model for repairable items only. Each base has two options: accept the repairable items, or deliver the repairable items, but not both. If the base chooses the “accept” option, we make sure that that base has used all its available resources, initial inventoried items and repair capabilities, before it receives re-supplies. In the transportation/delivery submodel, we place limitation on the crew duty hours available. The objective function is to satisfy the demands by minimizing traveling cost and inventory cost.

We solve the same problem by using a generalized Benders decomposition technique. The decomposition allows us to attack larger problems and use general failure and repair functions familiar to logistics analysts. Computational experience suggests

that efficiency is achieved by combining the delivery and inventory functions. Lateral re-supply and computational efficiency is particularly useful in emergency situations.

A COMBINED INVENTORY AND DELIVERY MODEL FOR REPAIRABLE ITEMS

Chapter 1-Problem Background

Overview

Is logistics a one-time a life thing? Is it ending? Is it crucial? Is it possible to think of a private or public enterprise that does not need to consider the logistical support?

I would like to answer these questions by pointing out the importance of logistics with a Turkish quote: “No matter how much an instructor knows in his area, he is a good instructor only as much as his students comprehend the material.” It seems there is no relation between this quote and logistics, but I will clarify the relation later in this section. If the capacity of a student is limited, then you should plan and apply the education program, and finally evaluate the student’s performance by placing an exam accordingly.

Think about a Dessert Storm / Dessert Shield scenario. Instructors correspond to Staff Commanders from all countries attending this operation, and students correspond to operational, logistical, and medical units, etc., that are going to execute the plan. It takes some time to be ready for the exam that corresponds to real operations. Students are provided with pens, pencils, books, notebooks as well as additional class notes. Instructors ask some basic questions to help them understand the material, and assign some homework and projects to increase the readiness level for the exam. Assuming students will pass the exams can mislead us, because we are skipping the time process

and the availability of materials to get ready for the exam. If the students are responsible for exams, we should provide them with the materials they need and time to study for the exam.

When we go back to the Dessert Storm / Dessert Shield Operation example, we assumed that all units are trained under operational conditions. It means that any unit can respond to changing situations very quickly and efficiently in terms of answering questions as a student during class. They can get the mission done with the provided equipment in terms of turning in projects, homework and passing exams as a student.

As a result, all students are good enough as long as the instructors know their student's capacity and let them to study for exams with the appropriate tools. We cannot assume that all students will pass the exam without time to study and the appropriate tools to apply. That is the reason the United Nations (UN) brought logistical materials first, months before bringing any operational units. Here we can relate this situation to our example as follows. Instructors that correspond to UN staff personnel provide the materials for students that correspond to any kinds of operational units.

We find out that logistics is crucial by just looking at the preparation time. The instructor and student interaction is forever while they are in education process. This is true for logistics as Bowersox et al. states out that logistics is unique: it never stops! Logistics is happening around the globe, twenty-four hours of every day, seven days a week during fifty-two weeks a year. (Bowersox and others, 1996:3)

Supply Chain Management

If logistics is a crucial, important, on-going process, then what is the definition of it? We can see a lot of titles used for logistics partly, or completely in the literature such as business logistics, physical distribution, materials logistics management, materials management, physical supply, logistics of distribution, marketing logistics, inbound logistics and total logistics. In 1991, Bowersox et al. states that

the Council of Logistics Management modified its 1976 definition of physical distribution management by first changing the term to logistics and then changing the definition as follows: "Logistics is the process of planning, implementing and controlling the efficient, effective flow and storage of goods, services and related information from the point of origin to the point of consumption for the purpose of conforming to customers requirements." (Bowersox and others:4)

After we defined logistics, we can figure out the relationship between logistics and supply chain management by looking at the definition Chan, et al. presents:

Integrated logistics, or supply chain management (SCM) is the integration of all key business processes from end-user through the original suppliers that deliver the right products to consumers at the lowest possible time and cost. The elements of supply chain management such as cost, quality, timeliness and flexibility should be accomplished in every phase of the supply chain management. (Chan and others, Sustainment Logistic Proposal:1).

SCM allows us to deal with different ways of implementing logistics such as logistics planning, supply, production, distribution, transportation, inventory etc. at the same time in contrast to earlier conceptions of logistics. Bowersox, et al. lays down this change in the conception of logistics as a paradox. Logistics has been performed since the beginning of civilization: it is hardly new. However, implementing best practices of the integration of information, transportation, inventory, warehousing, materials

handling, and packaging, etc. has become one of the most exciting and challenging operational areas of business and public sector management (Bowersox and others:4).

We can find another definition of SCM reflecting the production aspect of the business in the literature. Dornier et al. calls SCM the management of activities that transform raw materials into intermediate goods and final products, and that deliver those final products to customers (Dornier and others: 1998).

Statement of the Problem

We have node 0 (main depot) and, nodes 1, 2, 3 (3 bases) and 2 vehicles. In the beginning of the period, each base depot has an initial inventory. There is a minimum and maximum inventory level for each base node. Failure parts arrive at the bases with a Uniformly distributed mean λ_i ($i=1, 2, 3$). Every node have a repairing capabilities with an Uniformly distributed service rate μ_i ($i=0, 1, 2, 3$).

We will schedule the vehicles whenever the inventory level goes below its minimum level to minimize costs such as inventory cost, and shipping cost. The inventory cost is uniformly distributed in every node. In other words, the shortage and surplus cost is the same regardless of the number of the level of the demand.

There is a cost associated with inventory such as holding, shortage and surplus cost. There is a fixed cost associated with repairing the item in every node. There is a cost associated with using vehicles for shipping serviceable parts in the inventory from any node_{*i*} to node_{*j*} where $i, j = 0,1,2,3$.

We assumed that we have one type of cargo. We can re-supply from any node to another within the system. But we can only provide direct shipments between nodes.

We assumed that we know the steady state failure rates associated with each base, and steady state service rate associated with each node (main depot and three bases). In addition to that we know the shipping cost from node i to node j in terms of time where $i, j = 0,1,2,3$. We know the initial inventory level, maximum and minimum inventory level for each base depot and main depot as well.

We decide which vehicle to use and how many parts to deliver from node i to node j including both repaired items at repaired shops and inventoried items at depots for each node.

Background

When we examine the definition of logistics, we see that it consists of several areas. It is in the industrial area while producing goods. It is in the inventory area after we produce but before we transport the goods from the production plant (depot) to the customers (demand points) stationed at various points. Initial inventory position, maximum inventory position which is equivalent to the depot/base storage capacity, and distribution of the demand at each demand point as well as various kinds of costs such as holding cost, surplus cost, and shortage cost are all important for both inventory and transportation issues. Sherbrook asks some important questions such as “How can we insure that 95% of our scheduled aircraft flights will not be delayed for lack-of spare parts?” Additionally he asks some more supplementary questions like what would be the optimal level in order to answer this question satisfactorily. More generally “What can we do to change our logistics support structure to achieve a desired availability more efficiently?” Or “Is it economical to have more

repair capability at the operating sites?"(Sherbrook, 1992:2). All these types of questions are important when we examine the problem in the inventory area.

It is in the transportation area –or more specifically vehicle routing area if we decide to ship the goods with a fleet of vehicles to the demand points provided that there is a need to be visited by any vehicle due to the lack of inventory level. Various time windows associated with each demand point can restrict delivery times.

When we add the routing component into the problem, it becomes a routing-inventory problem. Transportation requires some consideration about the inventory level of a demand point. Stenger explains the tradeoff between inventory and transportation in his seminar held at the University of Cincinnati as follows. As we do things to reduce transportation costs (bigger shipment sizes and/or slower, less reliable modes), we increase the need for inventories (cycle stocks and safety stocks) at the receiving point (and possibly at the shipping point) (Stenger, Seminar at UC). It is of great of interest if we examine these two areas together. We generally try to answer the questions as F.Baita et al. explain in their paper.

When shipments have to be made; i.e. when trucks have to be loaded and when customers have to be visited; how much each truck has to be loaded, in terms of quantity of each item under consideration, and how such a load has to be distributed between the requesting customers; which route has to be followed by each truck in order to visit its customers (Baita and others, 1998:586).

Almost all of the inventory and vehicle routing problems try to find answers for the questions above. There may be dynamic attributes associated with each demand point. In other words, the demand for parts can be unknown and subject to change due to prior decisions about the three main questions mentioned above. F. Baita et al. classify

dynamic routing-inventory (DRAI) problems in terms of seven main groups. These seven main groups are topology of the problem such as one-to-many, the number of items in the problem such as one item in our problem, the attributes of demand in the problem such that it can be known and lies between two boundaries like our problem, the type of constraints in the problem associated with trucks, stock, supply capacities, costs associated with inventory, repairing, and distribution, and finally solution approach to the problem such as mixed programming models including linear, non-linear and integer programming (Baita and others, 1998:587). We will cover each element in greater details in chapter two.

We will assume that the demands for parts will be unknown beforehand. We cannot know what kinds of malfunction in the aircraft will cause the pilot to return to base without completing his mission. Even though the demand for parts is not deterministic (known), we can take advantage of uniformly distributed steady state values for failure and repair as well as inventory.

In addition to that, repair comes into play when demand points have a limited repair capabilities as well as preventive and corrective maintenance over capital equipment which directly affect the various kinds of inventory and transportation costs. As McGrath states, capital equipment maintenance is more important if you have large factories, energy generation power plants, refineries, and organizations that operate large fleets of land, air, or seagoing vehicles (Mc Grath, 1999:31). We can think of Air Forces in terms of large factories that keep huge number of items and large fleets of air and land vehicles in inventory. In our case, flight units, customers, bring failed item, and request for a serviceable item. As long as we get the failed item, we want to repair it by using

local resources if it is possible. If it is not possible, we then take advantage of the main depot repair capabilities. Repair capabilities have two folds. One of them is repairing the coming failed items out of aircraft at the base repair shop, and the other one is repairing capital equipment such as land vehicles used for carrying personnel and material on, off-base, machines, tools and equipment used for production, testing and calibrating etc. We are concerned with the former one in our model.

When we include inventory, repair and vehicle routing with time windows we come up with a breadth representation of SCM. Notice that repair is a matter of military or more generally government sector rather than that of private sector. It is unlikely to see such an example that involves all three main areas. Although we do not want to impose that we have to accomplish it within one unique model, we want to go through a case that requires combining all of them in SCM with time windows. If this is the case, we can have a breadth understanding of the issue. When we consider the application of this issue on Turkish Air Force, we have an insight what would be advantages and disadvantages if we had a combined model as we developed in this thesis.

Scope and Limitations

I am going to scope the problem with respect to the nature of the problem. Keeping in mind that if we choose the problem very broad we cannot investigate the problem very deep. In other words, there would be a tradeoff between going breadth and depth. This criterion would be determined by the problem itself.

I will try to come up with a model including repair, inventory and transportation areas. The assumptions made in the model should match very close to Turkish Air Force applications. We can list these assumptions as follows.

- A. The model will deliver whenever bases demand for items.
- B. The model will use all available resources at base before placing a request for item.
- C. The model will use all available vehicles if possible.
- D. If there exists a delivery of an item into node i , then no delivery goes out of node i .
- E. The model will deliver and pick-up only “serviceable” items. We assumed that we run the model one time for a steady state.

The purpose of this model can be summarized in three steps. In the first step, we will analyze and comprehend the inventory/allocation model (Federgruen and Zipkin, 1984:1019). Here we take an advantage of a numerical example in Chan’s textbook (Chan, 1999-Draft: 9-6). In the second step, we will apply the generalized Benders decomposition technique to the problem as Federgruen and Zipkin did. Benders cut is playing a major role in the solution process. Since the Benders cut is composed of dual variables, we will perform a sensitivity analysis for the dual variables associated with main depot capacity and vehicle capacity constraints. We should come up with the same result as Federgruen and Zipkin did. It is summarized as follows: Combining vehicle routing and inventory should save a lot of resource in terms of time or money. In the third step, we extend the starting point model, which corresponds to our thesis model, in such a way that we let the model issue the lateral supply between every nodes in the

model. We track the items individually; namely, they may come from either inventory or repaired shop of the node. We will examine the results whether combining vehicle routing and inventory is useful in this context or not.

The stopping point would be a model related with all/some of the areas. After we would take advantage of Air Force Material Command Studies and Analysis Office and Turkish Air Force senior officers assigned at Wright Patterson AFB to validate the model we would perform the sensitivity analysis related to comparisons and relations in each area described above.

One of the biggest limitations for this study is to find a real data for failure arrival rate and repair service time. . The model assumes that we have enough capabilities and resources so that we have a very past data to obtain a mean value for the repairing process. We would make up the data to run the model. In addition to that I will not go through repairing process itself, as well.

Chapter 2 Literature Review

Introduction

This chapter presents a general overview of the literature, which is closely related to the application of vehicle routing, repairing, and inventory problems in SCM. We would explain the components of SCM first and then give a general terminology and basic principles related to each area.

The Relationship between Vehicle Routing and Traveling Salesman Problem

As we stated in chapter 1, the definition of SCM requires delivery requirements from origin(s) (depot) to destinations (bases) via a fleet of vehicles. We recall a very well known type of problem such as traveling salesman problem (TSP) when we are considering visiting bases. Since many scientists put their efforts into solving TSP, we can easily find a very clear and concise definition of TSP in the literature. According to Chan, if the origin is the same as the destination, the path is called a tour with the origin/destination as the home base or the depot. Such a tour is often referred to as a hamiltonian circuit or cycle. Generally, we want to find the shortest tour that visits all the nodes exactly once in a depot-based continuous tour. This is formally known as the TSP. (Chan, 1999Draft:8-16). Note that you can start your travel from an origin node, which can be any node in the tour, and come back to the origin node provided we visit each node exactly once in the tour. It means that we want to solve a TSP in order to come up with the shortest tour satisfying a minimization objective function that can be expressed

in terms of money, time, or any kinds of value we can measure. We have more than one traveling salesman in the generalized TSP.

We can take a look at TSP's in terms of a distance matrix. It would not take the same amount of time if a vehicle passes through city during rush hour rather than in the morning at two o'clock. This idea is valid for aircraft as well. Going from point A to a point B with the back wind would not be the same as traveling the same distance with the head wind. Chan points out this point as follows.

If the distance matrix is symmetrical, TSP does not depend on the direction of travel and every node has two arcs incident to it. For asymmetrical distance matrix, however, there is one arc entering a node and another out of a node specifically. The former is referred as a symmetrical TSP, and the latter an asymmetrical TSP. Computationally speaking, symmetrical TSP's are much easier to solve.
(Chan, 1999Draft:8-19).

It seems to us that nothing is wrong with the formulation and solution of TSP.

But complete enumeration takes a extremely long time as Reeves points out:

As starting point is arbitrary, there are clearly $(N-1)!$ possible solutions (or $(N-1)!/2$ if the distance between every pair of cities is the same regardless of the direction of travel). Suppose we have a computer that can list all possible solutions of a 20 city problem in 1 hour. Then using the above formula, it would clearly take 20 hours to solve a 21-city problem, and 17.5 days to solve a 22-city problem; a 25-city problem would take nearly 6 centuries.(Reeves, 1995:7).

Because of the exponential growth in the complete enumeration, we should be very careful in selecting the number of nodes in the problem.

We can define various kinds of problems originating from TSP. Chan gives a definition of the vehicle routing problem (VRP) related to TSP as follows. If the delivery

requirements are placed upon the various bases (demand points) of a TSP, we end up with a VRP.

LaPorte, Louveaux, and Mercure point out the similarity between multiple TSP and VRP as follows:

The classical VRP consist of optimally designing vehicle routes from one or several depots to a set of customers in such a way that:

- (i) All vehicles start and end their journey at the same depot
- (ii) All customers are served once by exactly one vehicle, but a vehicle route may include several customers
- (iii) Some side constraints on the routes are satisfied
- (iv) The sum of vehicle utilization costs and of routing costs is minimized (La Porte, 1989:71).

LaPorte et.al points out the assumptions for the classical VRP as follows:

- A. The demands of cargo at each destination are fixed.
- B. The starting /ending locations are fixed.(La Porte, 1989:72).

We can relax the third similarity in such a way that we do not send a vehicle unless there is a need at the bases. In addition to that we can relax the first assumption made in the classical VRP in such a way that demands can be defined by a random variable. These two relaxations make the classical VRP convenient for solving real world issues.

The Relationship between Vehicle Routing and Inventory Routing Problem

By the solution of TSP, we obtain an order to visit each demand points exactly once so that we minimize the cost associated with each traveling time between depot and demand points. If we turn it into VRP, then our objective function as Haughton et al.

points out is to find a set of delivery routes that simultaneously satisfies demand at each retail outlet (demand points) and minimizes total transportation cost (Haughton, 1999:25). Note that we add the delivery requirements to various demand points in VRP without considering inventory issues at the demand points. Dror and Ball give a definition for inventory routing problem (IRP) as follows.

The IRP involves a set of customers, where each customer has a different demand on each day. For example, each customer uses a commodity such as heating oil or methane at an estimated consumption rate. Each customer possesses a known capacity (for example, the size of his tank to hold home heating oil). The objective is to minimize the annual delivery costs while attempting to insure that no customer runs out of the commodity at any time. The impetus behind this distribution system is the importance of maintaining a sufficient supply of inventory at a customer's location (Dror, 1987:891).

When we examine the definition of IRP, we realized that we should consider the annual delivery as an objective function to minimize. We can use a long-term delivery or steady state delivery instead of annual delivery. We will mention a lit bit more on annual delivery later.

The demand of each customer for the IRP is small compared with the capacity of the vehicle. IRPs have inventory-related costs and incentive mechanisms for early delivery to a customer.

We see some other definitions of IRP in the literature under the name of allocation/routing or assignment routing problems (Dror, 1987:892).

We extend classical VRP by adding periodic horizon time in order to get a similarity to the IRPs. Christofides et al. explains the Periodic Vehicle Routing Problem (PVRP) as follows.

In PVRP the problem is to design a set of routes for each day of a given (p-day) period. Each customer may require a number of visits by a vehicle during this period. If a customer requires k(say) visits during the period, then these visits may only occur in one of a given number of allowable k-day combinations (Christodifides, 1984:237).

PVRP is similar to IRP in the sense of long-term period and satisfying customer demand. However, it is different in the sense of specifying the number of visits during the period. The decision-maker has a power to determine the number of visits during the period in PVRP whereas IRPs let the model decide the number of visits and the size of delivery.

According to Russel et al. we can consider the routing aspect of IRP as follows.

Routing design problem in which the objective is to assign customer demand points to days of the week in such a way that the resulting node routing problems yield a near-optimal solutions. It is assumed that demand points offer some flexibility in their assignment to days of the week; each point may require service anywhere from one to seven days per week. This routing design problem assumes that the resulting node problem on each day of the week is a single depot vehicle dispatch problem whose objective is to minimize the distance or time required to service customer demand points and to minimize the number of vehicles required (Russel, 1979:page 1).

Routing design problem (RDP) is similar to IRPs in the sense of assigning vehicles to demand points; however, since the service day of any node in the model is predetermined it is different from IRPs in that respect. The primary objective function of the model is to minimize total distance (time) traveled per week subject to some constraints very similar to classic VRPs. Demand points may require being visited one, two or up to six days of the week assuming six business-day in one week. The model decides the assignment of vehicles after demand is determined; namely, decision-maker

puts a control mechanism on the demand. This type of formulation is valid for either delivery or pick-up type problems such as refuse collection, scheduled retail and wholesale delivery problems as Russel et al. explain.

According to Federgruen et al., we can consider a combined vehicle routing and inventory/allocation problem very similar to IRPs. The authors point out this problem as follows.

There are many situations where the vehicle schedules and the delivery sizes are (or should be) determined simultaneously. Such is often the case, when at each location the demand for the resource is random. Here, deliveries serve to replenish the inventories to levels that appropriately balance inventory carrying and shortage costs, but thereby incur transportation costs as well (Federgruen, 1984:1019).

A combined vehicle routing and inventory/allocation problem is very similar to IRPs in terms of delivering items to demand points via fleet of vehicles over a period. As Federgruen states out this type of problem is valid for internal distribution problems such as delivering fuel oil to automotive service stations. All decisions are made centrally in an internal distribution problem. Besides that, the model can be applied to some external distribution applications such as gas producers themselves install tanks at customer locations and determine the replenishment frequency and delivery sizes as Federgruen states out as well (Federgruen, 1984:1019). However, decision-maker does not have any control mechanism on demand as Russel et al. have in this type of model. In addition to that, the model does not need requiring k -visits to node i per period as Christofides et al. do. The period can be day, week, month, or even a year in Federgruen's model. But Dror et al. state out some reasons not to consider one period as a year as follows.

1. The validity of a formulation with an annual time base is questionable because some parameter values are uncertain over that long a period, others change.
2. The number of constraints and variables for such a formulation is prohibitively high (Dror, 1987:891).

As a starting point of understanding the concepts of delivery models with inventory concern, we decided to solve a toy problem that includes a main depot, two capacitated vehicles, and three bases embellished with newsboy inventory (Chan, 1999 Draft: 9-6). The concepts and assumptions of the toy problem is the same as what Federgruen et al. cover in their studies as follows.

The initial inventory (perhaps supply remaining from the previous day) for each location is reported to the depot. This information is used to determine for the following day the allocation of the available product among the locations. The assignment of locations to the vehicles and the routes are set at the same time. After deliveries are made (say at the end of day) the demands occur, and inventory-carrying and shortage costs are incurred at each location proportional to the end-of-the-day inventory levels (Federgruen, 1984:1020).

We try to minimize the transportation and inventory cost in this toy problem.

The results show us that the total cost is driven by inventory cost. A small portion of the total cost is due to transportation costs. Therefore, we place a close examination into the inventory aspect of the issue.

There are a lot of delivery models integrated with inventory concern in the literature as well as Baita et al. and Herer et al. cover in their studies, respectively. We will touch the routing-and-inventory models in great detail in the following section.

Dynamic Routing-and-Inventory problems

Baita et.al. present three basic aspects such as routing, inventory, and dynamicity that we can embed in VRP and IRP to form dynamic routing-and-inventory problems (DRAI). It includes routing because VRP requires organizing the physical movement of goods between different geographic sites, such as depots, warehouses, production and retail points, etc. It includes inventory because elements such as quantities and values of the goods being moved are relevant to the definition and assessment of organizational and operational strategies. It has a dynamicity property because routing and inventory are imbedded in a dynamic environment. In other words, repeated decisions have to be taken at different times within some time horizon, and earlier decisions influence later decisions (Baita, 1998:585).

Table 1: The elements for a classification of DRAI.

Classification element	Attribute	Alternatives
Topology	Endpoints	One-to-one, Many-to-many
Items	Number	One-to-one, Many
Demand	Knowledge Time behavior Distribution	Known, uncertain Constant, variable Uniform, not uniform
Decision type	Domain	Type, frequency
Constraints/objective	Truck capacity Stock capacity Supply capacity Vehicle number	Equal, different Given
Costs	Inventory Transportation	Holding Fixed
Solution approach	Decomposition Clustering Model / Algorithm MP models	Time, Cluster route Time, frequency Exact, approximate LP, IP, NLP

After we lay out the general attributes of DRAI, we would like to explain the classification elements of DRAI problems that we presented in chapter one.

The first classification element is topology, which can be one-to-one, one-to-many, or many-to-many. For example; one-to-many denotes that we have one origin (depot or distribution center) and many destinations (demand points or bases).

The second one is number of items we consider can be a single item or several. F. Baita et al. points out the importance of dealing with several items independently. This idea may be an important even though we have one item in the problem setting as Bauer et al. point out. Here we assumed that one item corresponding to one type of item and multi-item corresponding to multi-type items. While we are studying the problem that

Bauer et al. presented like a part of our thesis problem, we see that we should track the items independently. For instance, we may want to know what percentage of items delivered from depot and demand point 1 to demand point 2 if we assumed that the vehicle 1 follows the route in the sequence of depot-demand point 1-demand point 2. Vehicle 1 pick-ups some items out of main depot to deliver node 1, pick-ups some items out of node 1 to deliver to node 2. Besides that there is another possibility that vehicle 1 pick-ups some items out of main depot to deliver node 2 or pick-up out of node 1 to deliver main depot. All these possibilities are valid for delivery/pick-up problems as Bauer et al. solved in their working paper. When we consider the following situation in terms of aircraft parts, it is not acceptable to deliver to node 1 and pick-up out of node 1 simultaneously on the same tour. For example, if we deliver out of main depot to node 1, then we cannot deliver out of node 1 to node 2 or main depot. It is clear that the desire to deal with items independently will cause some difficulties as the problem size grows.

The remaining properties of DRAI problems such as demand, decision type, constraints/objective function, and costs excluding the solution approach are the basic among inventory models as well as Tersine state out (Tersine, 1994:12). We will cover these along with DRAI problems.

The third one is the characteristics of demand that consist of three elements like knowledge, time behavior and distribution. Since demand is a major element in DRAI problems and inventory models, we will cover this point in depth. When we say knowledge we mean that the demand can be either known or uncertain. It seems clear if we have a known demand. We can view the uncertain demand either as a random variable associated with an upper and lower bound or unknown completely. Demand can

be a constant value or random value across the time horizon when we talk about time behavior of demand. Baita et al points out the distribution property of demand such that it can be equally (uniformly) distributed or not when several customers or several items are involved. We note that we may not have a uniformly distributed demand even though we consider one type of items. For instance, the distribution function related to demand points could be Poisson, Exponential etc.

We can look at the demand in such a way that Tersine does. Demands can be examined according to their size, rate, and pattern (Tersine, 1994:12). Demand size is basically what we need/order in terms of quantity. It is important because it specifies the nature of the problem like deterministic model or stochastic model. Then the problem boils down to a point whether we know the demand size or not. Taha specifies the distinction as follows:

In the deterministic case, it is assumed that the quantities needed over subsequent periods of time are known with certainty. This may be expressed over *equal* periods of time in terms of known constant demands or in terms of known variable demands. These two cases are referred to as *static* and *dynamic* demands, respectively. Probabilistic demand occurs when the requirements over a certain period of time are not known with certainty but their pattern can be described by a known probability distribution. In this case, the probability distribution is said to be either *stationary* or *nonstationary* over time (Taha, 1976:390).

We know that probability distributions can be discrete or continuous. As Wackerly et al. state that the nature of the distribution function associated with a random variable determines the variable is continuous or discrete (Wackerly, 1996:136). We will go from backward to understand the distribution; namely, first define what are continuous or discrete random variables and then the distributions associated with them. A random variable that can take on at most a countable number of possible values is said to be

discrete (Ross, 1997:25). Wackerly et al. defines the probability distribution associated with discrete random variable as follows: The probability that Y takes on the value of y , $P(Y=y)$, is defined as the *sum of the probabilities of all sample points in S* that are assigned the value y (Wakerly et al., 1996:77). It means we can assign nonzero probabilities to only a countable number of distinct y values. On the other hand, the random variable whose set of possible values is uncountable is said to be continuous (Ross, 1997:31). Wackerly et al. defines the probability distribution associated with continuous random variable as follows: Let Y denote a random variable with distribution function $F(y)$. Y is said to be *continuous* if the distribution function $F(y)$ is continuous, for $-\infty < Y < \infty$ (Wackerly, 1997:139). As an example, demand for an aircraft part cannot be a fractional value that corresponds to discrete random variable such as Poisson and probability distribution function of Poisson whereas demand for repairing the failure item cannot be restricted to an integer number that corresponds to continuous random variable such as Exponential, Weibull and continuous probability distribution function of Exponential etc. As Tersine states out that we generally use standard probability distributions such as Normal, Exponential, and Poisson for demand (Tersine, 1994:12). As a result, if we know the demand size with certainty, the inventory model is deterministic; otherwise, it is probabilistic.

After we discuss the demand size, we can explain the demand rate based upon demand size. Demand rate is the demand size per unit of time. Demand pattern is corresponding to the pattern that how we are pulling the items out of inventory such as at the beginning of the period, at the end of the period, uniformly throughout the period.

The fourth one is the decision type involves two elements like frequency domain and time domain. Baita et al. point out that frequency domain decision type is appropriate with synchronous, periodic operations. In other words, decision variables are replenished between frequencies. On the other hand, time domain decision type is appropriate with asynchronous operating models, possibly using feedback (Baita and others, 1998:586-587). In other words, we can explain it as follows. The schedule of shipment, the quantities and routes are decided at fixed time intervals. Once we fixed them at the beginning of the period, we keep them the same until the end of that period. Herer et al. points out this difference related to his literature review in the paper as follows. The VRP is solved for a single period with known demands that correspond to time domain decision type, whereas inventory routing problem (IRP) is solved over several periods with stochastic demands that correspond to frequency domain decision type (Herer, 1997:71).

We can look at the difference as Tersine does as well. He uses the term replenishments for decision type. Replenishments can be examined according to size, pattern, and lead-time (Tersine, 1994:13). Replenishment size is basically the quantity or size of the order to be received into inventory, which can be either constant or variable depending on the type of inventory model. If the organization uses deterministic type model then the replenishment size is known, on the other hand if the organization uses stochastic type model, then the replenishment size becomes variable. The replenishment pattern is all about the way we added the replenishment sizes into inventory. It can be instantaneous, uniform, or batch. We should define first the term lead time in order to explain replenishment lead-time. The definition of lead-time is the time between placing

an order and its receipt in the inventory system as Anderson et al. define (Anderson and others, 1994:503). Replenishment lead-time is the lead-time of replenishments. It can be constant or variable the same as demand. We should use probability distribution functions to define the variable replenishment lead-time.

The fifth one is the decision variables involved in constraints and objective function such as vehicle (truck-airplane) capacity, stock capacity associated with demand points, and supply capacity related to origin points. We may have either homogeneous set of vehicles or not. In addition to that the number of vehicles we have can be a decision variable or it is known beforehand. We usually assume that we know the capacity constraints related to supply points, demand points and vehicles. Herer et al. classifies the decision variables in a three ways as follows. In the strategic level, the main decisions concern the location of facilities like plants, warehouses, and demand points. In the tactical level, the main decisions concern the fleet size and its attributes such as vehicle capacity. In the operational level, the main decisions concern the routing and scheduling of the vehicles in order to service the customers with corresponding quantities to be delivered (Herer, 1997:69). Since we solve our problem in the operational level, we try to decide which vehicle to use and how many parts to deliver from node i to node j including both repaired items at repaired shops and inventoried items at depots for each node. In other words, the number of vehicles, the locations of depots are given in our problem. As Tersine states out that the organizations may have some constraints such as management policies and administrative decisions as well as space constraints related to vehicle, depot, and personnel, capital constraints. Management policies such as never being stock out on certain items, and administrative decisions such as reciprocal

purchasing agreements can be another example to the type of constraints we have in the inventory models (Tersine, 1994:13).

The objective function of the inventory models is what to purchase or manufacture, when to take action, and in what quantity as Bowersox et al. point out (Bowersox et al. 1996:250). By assuming that we know what to keep in the inventory; namely, skipping what to purchase or manufacture stage, we can revisit the objective of inventory models as follows. The objective function of inventory models is to have the appropriate amounts of materials in the right place, at the right time, and at low cost as Tersine states (Tersine, 1994:13). Notice that the minimization of cost is not the sole objective in the inventory models. The objective is to minimize the inventory costs in the deterministic models whereas the objective is to minimize the expected cost of inventory in the stochastic models (Tersine, 1994:210).

The sixth one is the cost related to transportation and inventory. We usually minimize the cost in the objective function of DRAI problems provided we satisfy the demand. Inventory cost can occur in holding the items in the depot in the form of holding cost. It occurs in stocking the items less than or greater than the demand in the form of shortage cost, and surplus cost, respectively as well. We can add the fixed ordering cost to the objective function in order to be minimized. Setup cost is another issue to consider in considering the repairing an item at the facility.

Before going any further, I would like to define the various types of costs within the inventory models. Anderson et al. states the definition of holding cost as follows. It is the cost associated with maintaining an inventory investment; holding cost includes the cost of the capital investment in the inventory, insurance, taxes, warehouse overhead, and

so on. It can be interpreted as the surplus cost. If the demand is less than what we have then we will come up with the surplus cost. Order cost includes such items as making requisitions, analyzing vendors, writing purchase orders, receiving materials, inspecting materials, following up orders, and doing the processing necessary to complete the transaction (Tersine, 1994:14). Setup cost concludes the fixed charge associated with the placement of an order or with the initial preparation of a production or repair facility (Taha, 1976:389). We can think of setup cost and order cost as an independent of demand size or replenishment size. As soon as we place an order, those costs will occur. Shortage (stockout) cost is the penalty cost running out of stock when there is a demand for it. There exist two cases in situation of shortage as Taha states out as follows:

They generally include costs due to loss in customers' goodwill and potential loss in income. In the case where unfilled demand can be satisfied at a later date (backlog case), these costs usually vary directly with both the shortage quantity and the delay time. If the unfilled demand is lost (no backlog case), shortage costs become proportional to shortage quantity only (Taha, 1976:390).

We can even extend the shortages with respect to origins. It can be internal shortage where an order of the department within the organization is not filled. By the same token, it can be external shortage where customer order is not filled. We should recall the well-known problem in the literature named newsboy problem when we consider shortage and surplus cost all together across various levels of demands.

The transportation costs can show a wider variety as Baita et al. points out. They can have a fixed component per trip in particular, possibly depending on the load level of the trucks, plus variable components such as distance the vehicles traveled, and the number of customers visited (Baita, 1998:587).

The seventh one, the last one, is the solution technique to the DRAI problems. We can solve these problems as treating them as two sub-problems. Herer et al. points out these sub-problems as follows. Finding the optimal assignment of customers to trucks corresponding to clustering process, and finding the best route corresponding to route process (Herer, 1997:70). Baita et al. presents two decomposition of time horizon either of the type cluster-first, route-second, or vice versa. The second concern related to solution technique is how customers are aggregated into routes based on their geographical position, or on their estimated visit frequency, or on the estimated time to run off (Baita, 1998:587). What kinds of algorithms should we use for the solution? We have two choices; it can be either exact or approximate algorithm. If we decided to use any mathematical programming (MP) for the formulation, we may come up with the situation such that we may choose our solution technique accordingly based on characteristic of the problem such that it can be linear, integer, non-linear, or even mixed integer (Baita, 1998:587). It would be good to present the basics of these problems and the relationships between them. Winston explains the linear function and linear problems as follows:

A function $f(x_1, x_2, \dots, x_n)$ of x_1, x_2, \dots, x_n is a linear function if and only if for some set of constants c_1, c_2, \dots, c_n

$$f(x_1, x_2, \dots, x_n) = c_1x_1 + c_2x_2 + \dots + c_nx_n$$

For any linear function $f(x_1, x_2, \dots, x_n)$ and any number b , the inequalities $f(x_1, x_2, \dots, x_n) \leq b$ and $f(x_1, x_2, \dots, x_n) \geq b$ are linear inequalities. (Winston, 1994:53).

For example, $f(x_1, x_2) = 2x_1 + x_2$ is a linear function but $f(x_1, x_2) = x_1x_2$ is not linear function, it is non-linear function. $2x_1 + x_2 \leq 5$ is an example for linear inequalities,

$x_1, x_2 \leq 2$ is an example for nonlinear inequalities. Linear problems consist of linear objective function, linear equalities and linear inequalities and sign restricted signs that can be whether nonnegative any value such as integer and continuous, or unrestricted in sign. The integer programming problems is the same as linear programming problems except the restriction on the values of variables. An Integer Programming (IP) in which all variables are required to be integers is called pure integer programming problems and an IP in which only some of the variables are required to be integers is called a mixed integer programming problems (MIP) (Winston, 1994: 464). A nonlinear programming problem in which the objective function or some of the constraints may not be linear (Winston, 1994:639).

We have not talked about dynamicity attributes of routing-inventory problems. Dynamicity of demand is much more important than that of vehicles or customers in DRAI problems at the operational level.

When we consider the demand, which is a major concern of our interest, we would analyze it as Baita et al. state out:

In order to apply any stochastic model to a concrete real situation, a preliminary analysis of the demand is generally required, which in turn must be based on at least four elements:

1. A suitable large database of historical records.
2. Appropriate assumptions about the future behavior of the demand.
3. Appropriate assumptions about the form of stochastic processes that could suitably represent the demand.
4. The estimation, using statistical tools of the parameters of those stochastic processes (Baita, 1998:596).

As Baita et al. state out, it is not easy to carry out these steps from a practical point of view due to the stochastic assumptions that could adversely affect the possibility

of obtaining a model which completely adheres to the actual elements of the real situation under study. Because of the above reasons, Baita et al. states out:

Unknown demand, which only requires to know the bounds within which it could lie, seemed a sensible way to get more manageable models from a practical point of view, at least in some situations, as far as the process of assessing the demand is concerned (Baita, 1998:596).

An Overview to Inventory: Definitions

I would like to present a definition of inventory and to whom inventory is important. We can use the inventory in different meanings as Tersine states out as follows.

1. the stock on hand of materials at a given time (a tangible asset which can be seen, measured, and counted);
2. an itemized list of all physical assets;
3. (as a verb) to determine the quantity of items on hand;
4. (for financial and accounting records) the value of the stock of goods owned by an organization at a particular time (Tersine, 1994:3).

We would use the term inventory refers to the first definition. Tersine present a generalized definition of first one as follows. Inventory as material held in an idle or incomplete state awaiting future sale, use, or transformation (Tersine, 1994:3).

After we lay down the several definitions of inventory, we are ready to explain the organizations consider the inventory function as a part of their daily life. As Tersine presents that

The problems of inventory do not confine themselves to profit-making institutions but likewise are encountered by social and nonprofit institutions. Inventories are common to farms, manufactures, wholesalers, retailers, hospitals, churches, prisons, zoos, universities, and national, state, and local governments. Indeed, inventories are also relevant to the family unit in relation to food, clothing, medicines, toiletries, and so forth(Tesine, 1994:3).

Since it is widely concerned, One wonders that why all of the organizations above are keeping the same type of inventory?

Types of Inventory

We have four types inventory like supplies, raw-materials, in-process goods, and finished goods as Tersine states out. Supplies are inventory items consumed in the normal functioning of an organization. Supplies are not a part of the final product. We can give some examples to supplies such as pencil, paper, disks, facility maintenance items. Raw-materials are items purchased from suppliers to be used as inputs into the production process. Lumber, glue, screws are good examples of raw-materials to furniture manufacturer. In-process goods are partially completed final products that are still in the production process. Finished goods are the final products, which are available for sale, distribution or storage (Tersine, 1994:4). As an example, manufacturers hold all types inventory while wholesalers and retailers hold finished products in their inventory.

Since inventory is important and organizations can handle different types inventory at the same time, one may wonder the functions of it in order to take control of it. We are concerning the finished items in our thesis model.

Functions of Inventory

When we examine the first definition of inventory, we see that it is all about the number of stock on hand at a given time. We may ask this simple question without thinking. Why do we have to keep some number of stocks on hand? Or by the same token, what are the reasons for existence of inventory? As Tersine states out that

inventory exists for the following reasons and he explains the functions associated with inventory (Tersine, 1994:4). Supply and demand frequently differ in the rates at which they respectively provide and require stock due to four functional factors of inventory such as time, discontinuity, uncertainty, and economy factor.

Time factor of inventory involves the long process of production and distribution required such as time to develop production schedule, ship raw materials from suppliers before goods reach the final customer.

The discontinuity factor of inventory makes us deal with dependent operations such as retailing, distributions, warehousing, manufacturing and purchasing in an independent and economical manner.

The uncertainty factor of inventory includes errors in demand estimates, variable production yields, equipment breakdowns, strikes, acts of God, shipping delays, and unusual weather conditions.

The economy factor of inventory let the organization use cost reducing alternatives such as bulk purchases with quantity discounts instead of ordering separately without regard to transportation and lot size economies.

If we want to generalize what we covered so far, we find out that to determine the number of items on hand at a given time is a challenging task. Even though we are dealing with one type of inventory within the organization such as finished goods in retailer, we have four functions of it to consider. In our thesis problem, we are focusing on the time and uncertainty function.

Functional classification is another way to look at the inventory itself. It shows us a different aspect of inventory to consider. On the way to answering the reason of

existence of inventory, we covered the four function of it so far. We will touch on the classification aspect of inventory in the following section.

Functional classification

We can place all inventories in one or more of the following based on its utility as Tersine states and explains six functional classifications.

1. Working stock
2. Safety stock
3. Anticipation stock
4. Pipeline stock
5. Decoupling stock
6. Psychic stock (Tersine, 1994:7)

Working stock is inventory acquired and held in advance of requirements so that ordering can be done on a lot size rather than on an as needed basis.

Safety stock is inventory held in reserve to protect against the uncertainties of supply and demand.

Anticipation stock is inventory built up to cope with peak seasonal demand, erratic requirements (strikes, vacation shutdowns), or deficiencies in production capacity.

Pipeline stock is inventory on trucks, ships, and railcars or in a literal pipeline externally. On the other hand, if it is being processed, waiting to be processed, or being moved internally, then it corresponds to pipeline internally.

Decoupling stock acts as lubrication for the supply-production-distribution system that protects it against excessive friction. Remember that we can treat each system individually.

Psychic stock increases the chance an item is seen and considered for purchase. Full shelves increase sales whereas understock shelves as well as stockouts can lead to lost sales and lost customers.

Looking at inventory in terms of functional classification can let us focus on specifically one or more inventory class. We should manage every element of functional classification of inventory in such a different way that we should take consideration into their specific attributes. We can say that existence of inventory is based upon functional factors and functional classifications. We are dealing with safety stock and working stock classifications in the inventory side of the thesis problem.

After we figure out the existence of inventory, we may question ourselves in such a way that what kinds of problems we will encounter while determining the numbers items on hand at a given time. We will touch on the inventory problem classifications in the following section.

Inventory problem classification

We classified the DRAI problems in seven categories. Here, we will focus on just inventory problems. By putting side by side these two different categories, we will mainly explain the areas related to only inventory problems.

Tersine classify the inventory problems as follows. The inventory problems can be organized according to the repetitiveness of inventory decision, the source of supply, the knowledge of demand, the knowledge of the lead-time, and the type of inventory system (Tersine, 1994, 9-11).

According to DRAI problem classification, we cover the repetitiveness under the decision type section, the knowledge of demand under the demand section. One point that we did not consider about knowledge of demand in DRAI is independent and dependent demand. Demand for finished goods are characteristically independent while demand for raw materials, components and subassemblies are dependent.

The other classification elements are specific to inventory problems. We have two kinds of source of supply such as outside and inside supply. One unit of an organization supplies the items to the other unit within inside supply. On the other hand, outside resource supplies the items with a corresponding purchase amount within outside supply.

We can see the knowledge of the lead-time as a constant or variable. If it is variable, we may determine its distribution empirically or specified.

The last classification is about the type of inventory system. Some of the most common inventory systems are perpetual, periodic, material requirements planning, distribution requirements planning and single order quantity inventory system as Tersine explains (Tersine, 1994:11).

Perpetual inventory system orders stock whenever the inventory position reaches the reorder point.

The periodic inventory system orders stock at discrete points in time.

The material requirement planning (MRP) system orders stock only to meet pre-planned production requirements. Anderson et al. define it as follows. It is a computerized inventory system whose functions are to schedule production and to control

the level of inventory for components with dependent demand (Anderson et al, 1994:504).

The distribution requirements planning (DRP) system orders stock to meet distribution center requirements in multi-echelon network.

The single order quantity system orders stock to meet unique or short-lived requirements.

Relation between Inventory Problems and Newsboy Problems

As Winston pointed out the objective of basic inventory models is to minimize the costs associated with maintaining inventory and meeting customer demand by answering the following questions. When should an order be placed for a product? How large should each order be? The answer for the first question is related to demand function as well as shortage and surplus cost. The answer for the second question is related to vehicle capacity that carries the cargo from main depot to demand points, and base, depot capacity that keeps the cargo associated with a holding cost. On the other hand, the best way to comprehend shortage and surplus cost is to examine the well-known news vendor problem (NV). Gallego et al. defines the classical NV as follows.

The newsboy problem is to decide the stock quantity of an item when there is a single purchasing opportunity before the start of the selling period and the demand for the item is random. The trade-off is between the risk of overstocking (forcing disposal below the unit purchasing cost) and the risk of understocking (losing the opportunity of making a profit) (Gallego, 1993: 826).

In other words, we will have a shortage cost if the random demand is greater than order quantity. Alternatively, we will have a surplus cost if the random demand is less

than order quantity. Khouja states out relation between inventory and newsboy problems as follows. The single period problem (SPP), also known as newsboy or news-vendor problem, is to find the order quantity which maximizes the expected profit, or minimizing the expected costs of overestimating and underestimating demand, in a single period stochastic framework (Khouja, 1999:537). Remember the objective is to minimize the cost in a single deterministic framework. The relation between NV and inventory problems is always valid if we are dealing with a single period. One may wonder what the level of relation would be if we have multiple periods. Even though we used uniformly distributed demand in the newsboy inventory cost function, we want to lay down the possible extension out of NV. Since NV has wide applicability in economy, we can analyze the NV literature with more depth as Khouja does. The extensions to NV as follows:

1. Extensions to different objectives and utility functions
2. Extensions to different news-vendor pricing policies and discounting structures.
3. Extensions to different supplier pricing policies.
4. Extensions to random yields.
5. Extensions to different states of information about demand.
6. Extensions to constrained multi-product.
7. Extensions to multi-product with substitution.
8. Extensions to multi-echelon system.
9. Extensions to multi-location models.
10. Extensions to models with more than one period to prepare for the selling season.
11. Other extensions (Khouja, 1999:538).

We will mention these extensions roughly based upon Khouja literature review.

The ideas we presented under each extension are based upon several authors studies

covered by Khouja. We aim to give a general idea where we can extend the single period inventory problems and relate some of these extension to our thesis problem.

The first extension, extensions to different objectives and utility functions, says that maximizing the probability of achieving a target profit, denoted by P_B , is more consistent with the actions of organizations' managers than maximizing the expected profit. Deriving a closed form for an optimal order quantity, denoted by Q^* , for various kinds of demand distributions such as Normal, Beta, Gama is a matter of problem. The key point here Q^* provide us with the objective of maximizing profit and achieving P_B . The objective of achieving P_B can be extended under two cases. First case is all about shortage cost which can be either zero or greater than zero. Second case is a NV problem of two-product instead of one. Maximizing expected utility, and analyzing demand risk, which can be either risk averse or risk neutral, are another issue of objective function.

The second extension, extensions to different news vendor pricing policies and discounting structures, is about determining Q^* when suppliers offer quantity discounts. We can apply discount policy in three different ways. All unit quantity discounts are applicable for every item we purchased. On the other hand, incremental quantity discounts are applicable for items purchased after break point. Third discount type is about the level of loading items into truck. The more we close to the capacity of truck, the more discounts we have.

The idea of the larger container we used in transportation instead of smaller ones in NV the smaller units cost we have is true for the case where Q is made up of n standard size containers. Remember what Stenger said about the tradeoff between inventory and transportation. We will rephrase his idea for the completeness. As we do

things to reduce transportation costs (bigger shipment sizes and/or slower, less reliable models), we increase the need for inventories (cycle stocks and safety stocks) at the receiving point (and possibly at the shipping point) (Stenger, 1999). The key point in this context is simply we do not wait for demand to fill up to the capacity of vehicle as Stenger implies; instead, we have a chance to load the demanded item into larger or smaller containers. After we clarify these two ideas in order to show that there is no contradiction between two claims, we can touch on the situation where a proportion of customers are willing to wait for emergency supply provided we have a shortage of demand. This is another type for this context.

The third one, extensions to different supplier pricing policies, explains the relationship between customer demand and selling price of an item. The option can be decrease the selling price in order to increase the demand. The other option can be multiple discounts until the organization sells excess inventory.

The fourth one, extensions to random yields, considers the defective items in Q^* . By taking into consideration of these defective items, we can calculate Q^* under uniformly or exponentially distributed demand.

The fifth one, extensions to different states of information about demand, is all about parameters of demand function. It can be either simple demand distribution such as uniform or complex demand such that no known distribution can fit the demand function. Another promising option is more promising. It is sufficient to know the mean and variance of demand to be able to derive Q^* to maximize expected profit.

The sixth one, extensions to constrained multi-product, is focusing on multi-product instead of single product. When we think that a typical newsstand will have a

large number of products to sell, corresponding to n variables in the model, whereas a small number of constraints, corresponding to m constraints in the model. There is a possibility that we can convert the primal models into dual and solve the dual of it with fewer variables.

The seventh one, extensions to multi-product with substitution, is related to the following situation where there is shortage of demanded item. When a customer demands another item, if substitution his demand with another item, or goes to another newsstand to meet his demand.

The eighth one, extensions to multi-echelon system, is valid for selecting optimal components stock levels in an assemble-to-order system. As Tersine defines that when an order is received, the product is assembled from group standard subassemblies already in inventory. Since the product has numerous optional features and each customer may desire a unique configuration, finished goods are not usually available at the department. The assembly lead-time is usually quite short for most products (Tersine, 1994:24).

The ninth one, extensions to multi-location models, propose multiple locations in the problem setting. Each location has either same selling season or selling seasons of the different location lag each other. We may examine the former case as follows. we may have centralized system which consists of central depot and n retailer or decentralized system, which consists of independent sellers that keep inventory at their own expense. We can examine the latter case as follows. Khouja gives an example to explain this case such that the ending of summer season in United States of America may let the organization sell his remaining in Australia where summer is about to begin.

The tenth one, extensions to models with more than one period to prepare for the selling season, is related to selling items in more than periods in a single season. Here, we assumed that we have more than one period in the selling season. The problem we have in this type of extension can be in two fashions. The first one is determining the Q^* or production quantity towards the ends of season. The second one determining the probability that customer will buy the specified product in the beginning of season. As season rolls on, the probability of customers' demand to that item gets identified.

The last extension is not focusing on one area. Yield management can be an interest of this section. Khouja gives an example to clarify this type of management. Airlines offer discount-fare tickets if you buy the tickets before flight departures such as two, three weeks before. What would be the number of seats available at discount rate can be the one of the main questions in the yield management. Another example is advanced booking of orders. Buying seasons tickets before athletics games season started is an example for advanced booking of orders.

As a result of these extensions we see that NV is very applicable in both retailing and pure service organizations such as air transportation. Also, the reduction in product life cycles brought about by technological advances makes the NV more relevant (Khouja, 1999:550).

Without considering the distribution of demand function itself, it seems that the inventory problem can contribute a significant amount of cost to total cost due to various kinds of costs like holding cost, and shortage cost. This result brings our attention to a point such that we should take care of these costs in our formulation if we want to model real world issues.

The Relationship between Inventory and Repair Process

It is clear that inventory plays a key role in SCM. If we have high tech equipment like an aircraft in our inventory, then repairing process becomes as much important as the inventory. Because aircraft is consist of many high tech, high cost items (Sherbrook, 1992:xxi). Before going through inventory with the repairable items we should take a look at some more definitions such as reorder point, cycle time, and backorder.

Anderson et al. states these definitions in a clear way. Inventory position is what we have on hand plus the inventory on order. Reorder point is the inventory position at which a new order should be placed. Cycle time is the length of time between the placing of two consecutive orders. Backorder is the receipt of an order for a product when there are no units in inventory (Anderson, 1994:503-504).

By using the definitions above, it would be easy to comprehend the foundations of system-based repairable item inventory models that are currently used in United States Air Force (USAF). These principles are (S-1, S) ordering policy, Poisson processes, Poisson random variable, Palm's Theorem, and the nature of the demands (Kliger, 1994:9).

(S-1, S) Ordering Policies. If the inventory theory objective is to compute optimal stock levels for each item, then what is the stock level or inventory position? (Sherbrook, 1992:23). We should define the inventory position as follows.

$$(1) S = OH + DI - BO$$

where stock level denoted by S, which is a constant is equal to the number of units of stock on hand (OH), plus the number of units of stock due in from repair and re-supply minus the number of backorders (BO) (Sherbrook, 1992:23). Since repairable items tend

to be high tech, high cost, and low demand at a base, we can define the reorder point at a fixed stockage objective level like $S-1$. In other words, as soon as inventory level falls down by S , we place an order for an equal number of units that have been demanded (Klinger, 1994:10).

Poisson Processes.

We will explain some definitions to be able to understand Poisson processes. The definition and the properties of counting processes, the definition of independent increments are the key elements on the way to Poisson processes. A stochastic process $\{N(t), t \geq 0\}$ is said to be a *counting process* if $N(t)$ represents the total number of “events” that have occurred up to time t (Ross, 1997:249). The counting process must satisfy the following conditions:

- i. $N(t) \geq 0$.
- ii. $N(t)$ is an integer valued.
- iii. If $s < t$, then $N(s) \leq N(t)$.
- iv. For $s < t$, $N(t) - N(s)$ equals to number of events that have occurred in the interval (s, t) (Ross, 1997:249).

The only missing part in definition of Poisson process is independent increments. A counting process is said to possess *independent increments* if the numbers of events which occur in disjoint time intervals are independent (Ross, 1997:250). The number of events have occurred by time 5, that is $N(5)$, must be independent of the number of the events occurring between times 5 and 10, that is $N(10) - N(5)$, would be a good example for independent increments. By combining these three elements we would define Poisson processes as follows.

- The counting process $\{N(t), t \geq 0\}$ is said to be a *Poisson process* having rate λ , $\lambda > 0$, if
- i. $N(0) = 0$.

- ii. The process has independent increments.
- iii. The number of events in any interval of length t is Poisson distributed with mean λt . That is, for all $s, t \geq 0$ (Ross, 1997:251).

$$(2) P\{N(t+s) - N(s) = n\} = \exp\left\{-\lambda t \frac{(\lambda t)^n}{n!}\right\}, n = 0, 1, 2, \dots$$

The Poisson Random Variable.

A random variable X , taking on one of the values $0, 1, 2, \dots$ is said to be a *Poisson* random variable with parameter λ , if for some $\lambda > 0$,

$$(3) p(i) = P\{X = i\} = \exp\left\{-\lambda \frac{\lambda^i}{i!}\right\}$$

We can use Poisson random variable as an approximation of binomial random variable when the binomial parameter n is large and p is small (Ross, 1997:30).

Compound Poisson Processes.

We will define this concept and then give a complementary example for it.

A stochastic process $\{X(t), t \geq 0\}$ is said to be a *compound Poisson process* if it can be represented as

$$(4) X(t) = \sum_{i=1}^{N(t)} Y_i, \quad t \geq 0$$

where $\{N(t), t \geq 0\}$ is a Poisson process, and $\{Y_i, i \geq 0\}$ is a family of independent and identically distributed random variables which are also independent of $\{N(t), t \geq 0\}$. The random variable $X(t)$ is said to be a compound Poisson random variable (Ross, 1997:282).

The example for the compound Poisson processes, which are composed of many Poisson distributed random variables, can be as follows.

Suppose that buses arrive at a sporting event in accordance with a Poisson process, and suppose that the numbers of customers in each bus are assumed to be independent and identically distributed. Then $\{X(t), t \geq 0\}$ is a compound Poisson process where $X(t)$ denotes the number of customers

who arrived by t . Y_i represents the number of customers in the i th bus (Ross, 1997:282).

Palm's Theorem.

It enables us to estimate the number of items in repair at the steady state probability if we know the probability distribution of the demand process and the mean of the repair time distribution. The formal definition of Palm's Theorem is as follows:

If demand for an item is a Poisson process with annual mean m and if the repair time for each failed unit is independently and identically distributed according to any distribution with mean T years, then the steady-state probability distribution for the number of units in repair has Poisson distribution with mean mT (Sherbrook, 1992:21).

Nature of demand.

Since we talked about the demand in great detail under the DRAI problems, we would just say that we assumed the demand for repair items is independent. Demands can be deterministic or stochastic in repairable item inventory models as well.

Measurement Criteria.

We have three measurement criteria while evaluating repairable item inventory systems like fill rate, backorders, and availability.

Fill rate.

It is the percentage of demands that can be met at time they are placed (Sherbrook, 1992:24). The other definition for it is the probability that at least one spare item is available on the warehouse shelf when a demand for an item occurs; it is the probability that the number of demands during the re-supply time are strictly less than the spare stock level (Klinger, 1994:13).

Backorders.

They are the number of unfilled demands that exist at a point in time (Sherbrook, 1992:24). They are the numbers of “holes” in an aircraft, or the numbers of missing items on an aircraft (Klinger, 1994:13).

Availability.

Aircraft availability rate is the percentage of aircraft, which are available or fully mission capable (Klinger, 1994:13).

Integrated Logistics

Integrated logistic idea includes the idea of combining vehicle routing and inventory. The reason that we mention supply chain management in the literature review is to understand the importance of looking at individual processes together. After we present three main areas like vehicle routing, inventory and repair, we should come up with the following figure.

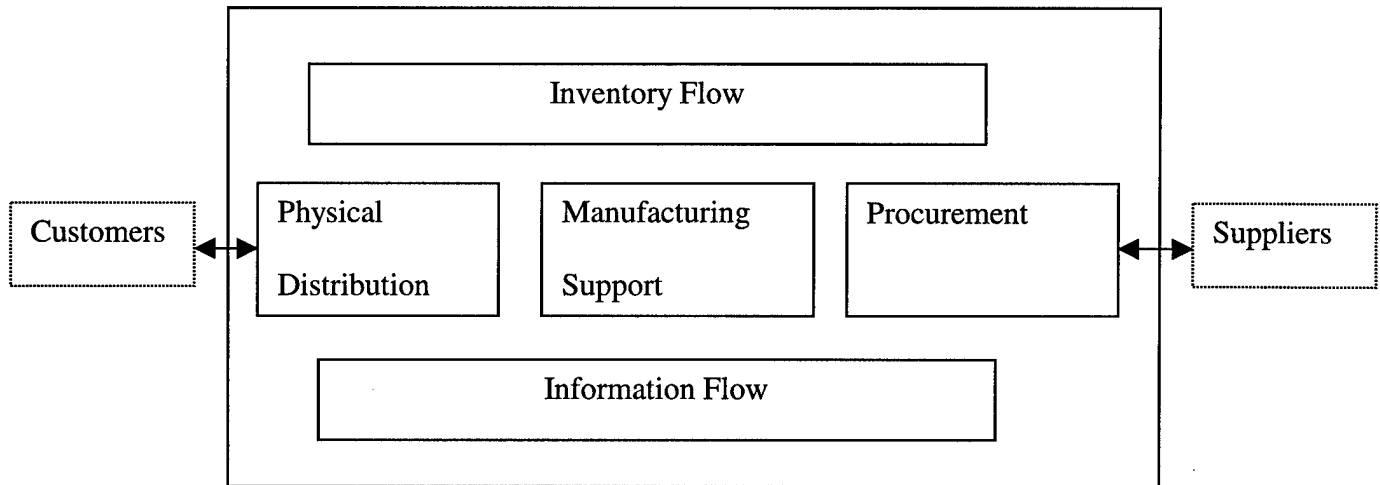


Figure 1: The supply chain management

When we look at the graph we see that inventory and information flow are passing through all three components. To be fully effective in today's competitive environment where a necessary condition for success is to have integration between internal functions, we should expand their integrated behavior to incorporate customers and suppliers (Bowersox, 1996:34). Inventory flow helps us to determine how many items, when and where needed (Bowersox, 1996:34). Information flow is composed of two parts: coordination flows and operational flows. The former one includes coordination between strategic objectives, capacity constraints, logistical requirements, inventory deployment, manufacturing requirements, procurement requirements, and forecasting. The latter one includes coordination between order management, order processing, distribution operations, inventory management, transportation and shipping, and procurement (Bowersox, 1996:37).

Physical distribution is concerning about movements of a finished goods. The overall concern of manufacturing support is not *how* production occurs but rather *what*, *when*, and *where* products will be manufactured (Bowersox, 1996:35). We are dealing with materials that are under the control of the manufacturing enterprise. The term procurement in government sector, purchasing in acquisition, or buying in general is concerns with purchasing and arranging in-bound movement of materials, parts, and/or finished inventory from suppliers to manufacturing or assembly plants, warehouses, or retail stores (Bowersox, 1996:35).

We have known that integrated logistics approach will yield a lower objective cost value. We have already known that Federgruen and Zipkin proved that combining the inventory and vehicle routing problems together would yield a lower

objective cost value. What we are trying to accomplish in Chapter Three is to show combining the vehicle routing and inventory in the new problem definition, which is extension of Federgruen and Zipkin model, will yield weather a lower objective cost value or not.

Chapter 3: Methodology

Introduction

The starting point of our thesis process is the problem (Federgruen and Zipkin, 1983:1020) which was presented in the literature review. First, we will list the parameters, variables, and sets of notations used for inventory/allocation and VRPs. Second, we will present an instance of this problem (Chan, 1999 draft:9-9). Third, we will show how to solve the starting problem either by using generalized Benders decomposition method or without using it. After we solved it without using generalized Benders decomposition, we will talk about the extension from the initial model including new variables, new parameters and set of notations. We will try to apply the generalized Benders' decomposition technique to the thesis problem in the last phase.

A Starting Point: VRP and Allocation/Inventory Problem

We will list the common variables and sets of notations to be used for formulating inventory and vehicle routing problems together as follows (Chan, 1999 draft:9-7):

x_{ij}^h is a binary variable equal to 1 if aircraft with tail number h flies the arc from i to j , and equal to 0 otherwise.

Z_i is a continuous variable that denotes the amount of allocation among available products to node i .

y_i^h is a binary variable equal to 1 if aircraft with tail number h allocates to node i and equal to 0 otherwise.

ξ is a random variable associated with a demand with any kinds of distribution.

H is the set of all aircraft where $h = 1, \dots, H''$ and $H'' = 2$.

I is the set of all nodes where $I = 0, 1, 2$, and 3 . The main depot is node 0 .

Additionally, we need to know the following parameters:

d_{ij} is the distance (in time units) between nodes i and j .

β_i is an initial inventory level of node i .

c_i is a unit carrying-cost or surplus cost function in location i .

C_i shortage cost in location i .

V_h is the vehicle capacity constraint associated with the aircraft with the tail number h .

\bar{P} is the amount of available resources at the central depot.

Chan lays down the model formulation explicitly as follows:

The objective function

$$(5) \text{ Minimize } \sum_{i \in I} \sum_{j \in I} \sum_{h \in H} d_{ij} x_{ij}^h + \sum_i q_i(z_i)$$

The objective function is to minimize transportation cost and inventory costs. The

inventory cost function is:

$$(6) q_i(z_i) = \int_{\beta_i + z_i}^{\infty} C_i(\xi - \beta_i - z_i) dF_i(\xi) + \int_0^{\beta_i + z_i} c_i(\beta_i + z_i - \xi) dF_i(\xi), \quad i \in I$$

where $F_i(\cdot)$ is the cumulative distribution function of demand occurred at node i .

We provide an example for the usage of the inventory cost function in the following sections in Chapter Three.

We will first lay down the sets of constraints found in the classical VRP and then, incorporate the additional constraints related to inventory in order to combine VRP and inventory/allocation problem. Here are the classical VRP constraints:

$$(7) \sum_{j \in I} \sum_{h \in H^r} x_{ij}^h = \begin{cases} |H^r| & \text{if } i = 0 \\ 1 & \text{if } i = 1, 2, \dots, |I| \end{cases}$$

$$(8) \sum_{i \in I} \sum_{h \in H^r} x_{ij}^h = \begin{cases} |H^r| & \text{if } j = 0 \\ 1 & \text{if } j = 1, 2, \dots, |I| \end{cases}$$

Constraint (7) and constraint (8) ensures that we used all available vehicles stationed at depot to serve the demand points.

$$(9) \sum_{i \in M_p} x_{ij}^h - \sum_{j \in \bar{M}_p} x_{ij}^h = 0 \quad \forall h, \forall p \in I.$$

Constraint (9) maintains the route continuity for all vehicle types where M_p denotes the out-of-nodes (source nodes) and \bar{M}_p denotes the into-nodes(sink nodes).

Note that M_p and \bar{M}_p are the same sets of nodes for each constraint.

$$(10) \sum_{j \in \bar{M}_0} x_{0j}^h \leq 1 \quad \forall h, \quad \text{and} \quad \sum_{i \in M_0} x_{i0}^h \leq 1 \quad \forall h$$

Constraint (10) consists of two constraints. They ensure that vehicle availability is not exceeded at depot 0 for vehicle h where M_0 denotes the out-of-node 0 and \bar{M}_0 denotes the into-node 0.

$$(11) \sum_{i \in L} \sum_{j \in L} x_{ij}^h \leq |L| - 1 \quad L \subseteq \{2, \dots, |I|\}, 2 \leq |L| \leq |I| - 1; \quad h \in H$$

Constraint (11) is the sub-tour breaking constraint where L is the non-empty set of I-1. The reason we are subtracting one out of the number of nodes in the problem is due to an having one central depot. All vehicles begin their tour from the main depot and

come back to it. This ensures that all tours in the solution be legitimate; namely, no cycle will occur in the solution.

The constraints related to allocation (inventory) side of the problem are now presented.

$$(12) \quad \sum_{i \in I} z_i y_i^h \leq V_h \quad h \in H$$

Constraint (12) ensures that the load assigned to each vehicle h is within its capacity.

$$(13) \quad \sum_{i \in I} z_i \leq \bar{P}$$

Constraint (13) guarantees that the total amount shipped is available at the depot.

$$(14) \quad \sum_{j \in M_j} x_{ij}^h - y_i^h \geq 0 \quad \forall i, h \in H \quad (\text{Chan, 1999 draft:9-7}).$$

Constraint (14) satisfies the linking constraints between vehicle routing variable x_{ij}^h and allocation variable y_i^h . If we place an order to node i , one arc out of node i should equal to one, otherwise zero.

A Numerical Example for the Starting Model

We mentioned in literature review that Federgruen and Zipkin combined the VRP and IRP in their study. For the sake of completeness, we would like to present the problem description again.

The initial inventory (perhaps supply remaining from the previous day) for each location is reported to the depot. This information is used to determine for the following day the allocation of the available product among the locations. The assignment of locations to the vehicles and the routes are set at the same time. After deliveries are made (say at the end of day) the demands occur, and inventory-carrying and shortage costs are incurred at each location proportional to the end-of-the-day inventory levels. (Federgruen, Zipkin, 1984:1020).

Johnson et al. and Moore et al. presented a four-node example for the problem of Federgruen and Zipkin which is covered in great detail in Chan's book (Chan, 1999 Draft:9-9). The problem is a combination of the shortest routing problems for multiple vehicles and the allocation/inventory problems.

The objective is to minimize the inventory costs incurring at each demand point and the travel costs. Inventory cost is the newsboy inventory cost where there are shortage and surplus costs. Travel cost is related to the distance traveled between nodes.

The objective function above is subject to the following constraints. There are maximum and minimum inventory levels for each demand point. We cannot deliver more than depot capacity. Available supply amount at the depot and the vehicle capacity constraint are additional limitations on the problem. We should assume that the vehicle capacity is not exceeded during the period. We are considering delivering only one type of supply materials to demand points, but not delivering and picking-up at the same time. Besides that we can visit each demand point with at most one vehicle without creating a cycle. We have two real vehicles and one dummy vehicle in the model. We used the dummy vehicle for two reasons. The first reason is to satisfy the TSP requirement, which is visiting each node exactly once. The second reason is to prohibit delivering to nodes which do not need any replenishments. In short, if the demand point does not need any replenishment, we send a dummy vehicle to that node so as to visit but not deliver. After the demand points declare the amounts at hand, the central depot prepares a delivery plan to minimize the objective cost, that is, to minimize the inventory and traveling cost for one period, which can be a day, a week, or a month.

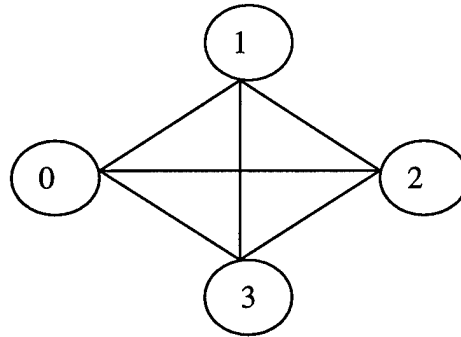


Figure 2 - A Starting Model - Single depot/multiple route/tour network

Node 0 denotes the main depot, and node 1, 2, and 3 denote the demand points in the basic network as shown above.

We displayed the cost matrix between each demand points and inventory allocation information in Table 3 and 4, respectively.

Table 2: Traveling cost matrix

	Node 0	Node 1	Node 2	Node 3
Node 0	-	3	3.5	2
Node 1	3	-	4	5
Node 2	3.5	4	-	3.8
Node 3	2	5	3.8	-

The traveling cost matrix is symmetric. Going from node i to node j is the as going from node j to node i . It can be interpreted as the time required from getting from demand point 1 to 2, or the cost associated with that traveling time. It is not a distance in terms of mileage, however, the distance is in terms of time. Even though the distance

between two nodes, let us say node i to node j , are the same, it takes different time units to get from node i to node j due to weather, air traffic, cargo load etc. Therefore, we preferred using time instead of mileage in the distance matrix.

Table 3 - Inventory-allocation data

Node	1	2	3
Max. Inventory	500	100	500
Min. Inventory	0	0	0
Initial Inventory	100	50	150
Demand (PDF)	0.002	0.010	0.002
Newsboy cost function	$2600 - 8z_1$	$437.5 - 7.5z_2$	$3012.5 - 6.5z_3$

On the other hand, we calculate the inventory cost function according to Uniform distribution. All the information we used in order to set up these two tables above is corresponding to parameter settings except demand in terms of probability distribution function (pdf) and newsboy cost function in the inventory/allocation data table. Note that the inventory cost functions is a nonincreasing functions. It means that the more we deliver provided that the depot capacity is available, the less we would have inventory cost. The second important observation in the inventory table is that there is no inventory cost function associated with main depot. Because there is no demand associated with main depot. The role of the main depot is to satisfy the demand. The third important

observation is that there is no lateral re-supply between nodes. It means that we can not supply node 1 from any demand nodes but only main depot.

A Look to the Inventory Cost Function at Demand Points

We would like to explain how we get the inventory cost functions (i.e. $2600 - 8z_1$) for demand point 1 by giving an example.

$$(15) \quad q_i(z_i) = \int_{\beta_i + z_i}^{\infty} C_i(\xi - \beta_i - z_i) dF_i(\xi) + \int_0^{\beta_i + z_i} c(\beta_i + z_i - \xi) dF_i(\xi), \quad i \in I$$

Let us $C_1 = \$20$ per item, which is shortage cost, $c_1 = \$5$ per item, which is unit carrying cost, $\beta_1 = 1$ unit, which is the initial level that is reported to main depot, and the depot maximum capacity is equal to 4 unit. By taking the integral of the inventory cost function with the given parameters, we would come up with the following result, that is $12.8 - 3.75z_1$.

After we understand how we develop the inventory cost functions, we can discuss the pdf's of the demands. We applied the pdf formulation of Uniformly distributed demand (Wackerly, Mendenhall, Scheaffer, 1996:150) which can be equally likely between zero and 500 unit for demand point 1, which corresponds to 0.002 in Table 4.

The Solution Approach to the Starting Point

We have two alternatives to solve the initial problem. One of them is to use generalized benders decomposition method, and the other one is to formulate and solve the problem without considering any issues. In the latter way, we let the solver pick the

appropriate switch on/off variables, y_i^h , that correspond to a placing order decision variables.

Generalized Benders Decomposition.

Benders partitioning procedure can be applied for solving mathematical programming problems that includes either integer, linear, a nonlinear in terms of variables or functions.

$$(16) \quad \underset{x,y}{\text{Min}} f(x,y) \text{ st. } G(x,y) \leq 0, \quad x \in X, y \in Y$$

where the functions f and g are differentiable function (Sahinidis, Grossman, 1991:481).

We can apply this technique for the following situations (Geoffrion, 1972):

For fixed y , (16) separates into a number of independent optimization problems, each of them has their own solution vector x . Multi-period design problems can be an example to this situation.

For fixed y , (16) yields a special structure that we can solve it efficiently. Mixed integer linear (MIP) and mixed integer nonlinear problems (MINP) can be for this condition.

Problem (16) is not in a convex region if we keep x and y all together in the model. But fixing y can lead us to have a convex set.

If you remember the starting point problem you will notice that there is a binary variable, y_i^h , such that we can fill the demand coming from the node if it is equal to one otherwise we cannot fulfill the demand if it is equal to zero. We realized that if we fix the placing order constraints, we would have a problem with a special structure that we can deal with a known algorithm.

Before going further, we will explain what we understand about fixing the placing order constraints. We know that we can visit the nodes at most one vehicle from problem assumptions. We have three nodes and three vehicles in the starting model, and each vehicle can visit each node. It means we can choose any vehicle to visit any node. For example, vehicle 0, which is a dummy vehicle has a chance to visit node 1, 2, and 3. This is true for the other vehicle 1, and 2 as well. It means we have three combinations for each node to visit, that corresponds to the combination of 27. Some of the combinations are certainly infeasible due to the fact that we should send two real vehicles out of and into the main depot.

As Geoffrion stated, we would have a MIP by fixing feasible y 's such that we would have two subproblems to solve in our starting model. These are a traveling salesman problem with k persons (k -TSP), and an inventory/allocation problem.

The original model written by Chan states:

Recall that $x^h = x_{ij}^h$ and $y^h = y_i$ are vectors of decision variables for routing and coverage respectively (Chan, 1999 Draft:9-8).

$$\text{Min } \sum_i \sum_j \sum_h d_{ij} x_{ij}^h + \sum q_i(z_i)$$

subject to

$$(17) \quad x^h \in X, y \in Y, h \in H$$

$$(18) \quad \phi^h(x^h, y^h) \geq 0$$

$$(19) \quad \sum_{i \in I} z_i y_i^h \leq V_h \quad h \in H$$

$$(20) \quad \sum_{i \in I} z_i \leq \bar{P}$$

where ϕ^h represents all linear inequalities defining the h^{th} TSP polytope. The set Y^h represents all the possible assignment of demands i to vehicle h provided

$$(21) \quad \sum_{i \in I} y_i^h = 1 \quad \text{for } \forall h.$$

Having presented the original model, we are ready to discuss the master problem, which is equivalent to the original model in y -space. The master can be stated as follows.

$$(22) \quad \text{Min } \{z \mid z \geq z'(y), y \in Y''\}$$

We should define the master Benders cut and the constraints associated with $y \in Y''$.

We obtained the Benders cut after a couple iterations in the following form. For those who are interested in the derivation stages, (Federgruen and Zipkin 1983: 1020), and (Chan, 1999 Draft, 9-8) are good references.

$$(23) \quad z \geq \Omega \bar{P} + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-0} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-0} (\kappa_i^h + \varphi_i^h) y_i^h.$$

Ω is the dual variable corresponding to the main depot constraint, and ρ^h is the dual variable to account for the h^{th} delivery-vehicle capacity constraints in the inventory/allocation subproblem. κ^h is the total travel cost on tour h , and κ_i^h is the unit arc cost (or marginal cost) to reaching node i .

φ_i^h is the inventory (newsboy) cost in the form of Eq.(24).

$$(24) \quad q(z_i) - (\Omega - \rho^h) \cdot z_i$$

These dual variables are playing a major role while forming the Benders cut. Both of them have an impact on the intercept and the slope of the Benders cut. If Ω is greater than zero, then the intercept point will be higher but slope will decrease. It means that we do not have a very deep Benders cut. Deep Benders cut means we divide the optimal region in such a way that we become so close to the optimal solution. We can

think of the same idea for ρ . If ρ is greater than zero, then the intercept point will be higher but the slope will decrease. In our numerical example, we reached the optimal solution within three iterations. We can perform a sensitivity analysis as well. For the first iteration the Ω is equal to zero, ρ associated with dummy vehicle and vehicle 1 is 8, and 6.5, respectively. It means if we increase the capacity of vehicle one by one unit, we improve the objective function by 6.5 unit. Larger vehicle capacity denotes higher intercept, but not steep slope. It means we can cut the optimal region mildly so that we can reach the optimal solution. In other words, it may take a couple of more iteration to get the optimal solution.

If both of them is zero, then we can generate many cuts without reducing the size of the optimal region. Since we have only operating cost to form up the Benders cut, we cannot generate a useful cut to reduce the feasible region. It means we cannot move toward the optimal solution. We stay at the same point in the same region, but the y_{ij}^h 's are changing.

After we define the Benders cut in the master problem we are ready to explain two sets of constraints associated with $y \in Y''$. The first set of constraint is the Eq.(21) that satisfies the condition that each node is visited by exactly one vehicle. The second set of constraints is equation (19) that satisfies the condition that we allocated vehicles within their capacities. Here we should note that since we are solving the master problem in terms of y 's, we must substitute the values of z_i 's found in the previous allocation/inventory subproblem into the Eq.(19). Solving the master problem in y give us a new vector of y 's to pass to k-TSP and inventory/allocation subproblem. We redo

the procedure until we generate no longer cuts for the master problem. The stopping criterion holds if we reach the point where the two successive iterations are all the same.

An example for the application of this procedure will help to clarify it. Let an initial feasible y_i^h be $y_1^0=y_2^2=y_3^1=1$ and the rest of y_i^h 's equal to zero. When we solve the k-TSP problem yielding the following result: $x_{03}^1=x_{30}^1=1$, and $x_{02}^2=x_{20}^2=1$, and $x_{01}^0=x_{10}^0=1$ with an objective function value of 17. The formulation and solution to the TSP sub-problem is at Appendix 1 and 2, respectively. By substituting the fixed y_i^h 's into the delivery allocation constraints (Eq.19), we solve the inventory/allocation subproblem in terms of z_i 's that yields $z_1=0$, $z_2=50$, $z_3=150$ with an objective function value 4700. The formulation and the solution to the Inventory/allocation sub-problem is at Appendix 3, and 4, respectively. The necessary values to set up the Benders cut are as follows. The dual variables associated with main depot supply constraint and delivery/allocation constraints for each vehicle is $\Omega=0$, $\rho^1=8$, $\rho^2=6.5$, and $\rho^3=0$, respectively. $\kappa^0=6$, $\kappa^1=4$, $\kappa^2=7$ are the operating cost for each vehicle. For example, vehicle 0 goes to node 0 and comes back, therefore its operating cost is $3+3=6$. We can calculate the other operating costs in the same manner. $\kappa_{13}^0=3$, $\kappa_{32}^1=2$, and $\kappa_{21}^2=3.5$ is the marginal cost servicing the node 1, 2, and 3, respectively. We give another example to explain this situation. If Vehicle 0 goes to node 1, its marginal cost to servicing node 1 is the cost where vehicle 0 comes from. In this solution set, $\kappa_{10}^0=3$. But if vehicle 0 visits node 1 after visiting node 2, then the marginal cost would be equal to the cost of going from node 2 to node 1, that is $\kappa_{10}^0=4$. By using the results of the two sub-problems, the Benders cut is

$$(25) \quad z - 5203*y_1^0 + 675*y_2^0 - 1675*y_3^0 - 2600*y_1^1 + 262.5*y_2^1 - 1065*y_3^1 \\ - 2600*y_1^2 - 66*y_2^2 - 2038*y_3^2 > 1000.5$$

and the y_i^h constraints are

$$(26) \quad 50*y_2^0 + 150*y_3^0 < 0 \\ 50*y_2^1 + 150*y_3^1 < 150 \\ 50*y_2^2 + 150*y_3^2 < 200$$

$$(27) \quad y_1^0 + y_1^1 + y_1^2 = 1 \\ y_2^0 + y_2^1 + y_2^2 = 1 \\ y_3^0 + y_3^1 + y_3^2 = 1$$

where y_i^h is a binary variable, Eq (26) is the vehicle capacity constraints, and Eq.(27) is the placing an order constraints for each demand point. The calculation of Benders cut is at Appendix 1F. The master problem above yields the new set of y_i^h such that $z=4731$, and $y_3^1=y_2^1=y_2^2=1$. The formulation and the solution to the relaxed master problem is at Appendix 5 and 6, respectively. We continue to solve the k-TSP and inventory/allocation problem with these y_i^h 's. We generate a new cut for the relaxed master problem and this process continues until the two successive iterations yields the same y_i^h 's and the same cut for the master problem. Table 4 shows the result of each iteration.

Table 4: The results of Generalized Benders Algorithm

y_i^h			z_i			operating cost	inventory cost	total cost
$Y_{01}=1$	$Y_{22}=1$	$Y_{13}=1$	$Z_1=0$	$Z_2=50$	$Z_3=150$	17	4700	4717
$Y_{13}=1$	$Y_{21}=1$	$Y_{22}=1$	$Z_1=200$	$Z_2=0$	$Z_3=150$	14.5	3475	3489.5
$Y_{12}=1$	$Y_{13}=1$	$Y_{21}=1$	$Z_1=200$	$Z_2=50$	$Z_3=100$	15.3	3425	3440.3

We may think of the solution process in a different manner. After we have fixed the y_i^h 's in the model, we will have a MIP. We may get a solution in terms of x_{ij}^h and z_i 's out of MIP while fixing the y_i^h 's. By the same token as we did in the previous solution method, we generate a Benders cut to solve the relaxed master problem which is equivalent to original one in terms of y_i^h 's. After we get the new set of y_i^h 's, we solve the new MIP with new y_i^h , and we form a new Benders cut and solve the relaxed master problem for the new set of y_i^h 's until we come to a point that we cannot improve the relaxed master problem objective function. At that point we assumed that we reached the optimal solution. I attached the first iteration of the MIP with fixed $y_1^0 = y_2^2 = y_3^1 = 1$ at Appendix 8. We got the same result as the one with Federgrun and Zipkin procedure in the first iteration as we expected. Since the result of the first iteration is the same as the previous method, it means we will get the same Benders cut for the following iterations. Therefore; we just show the first the result of MIP at Appendix 9.

In the first iteration we have no control on the solution due to the fact that we selected the y_{ij}^h 's arbitrarily. But in the second iteration, we come up with the y_{ij}^h 's such that the resulting TSP model yields the shortest tour with an objective function 14.5, and associated total cost including inventory/allocation objective value is 3489.5. In the third iteration, we assigned the vehicles to the nodes with a little bit slower assignment like 15.3 but the total cost goes down to 3440.3. This result shows us that combining vehicle routing and inventory yields better (lower) result than treating TSP and inventory/allocation individually.

Why do we Need an Alternative Approach to the Starting Point Problem

We used the Generalized Benders Decomposition technique to take an advantage of special structure of the problem and to solve it effectively in terms of computational time. But we know that we can reformulate the same problem as if we do not use any decomposition technique.

Firstly, since we want to extend the initial problem in several aspects, we want to formulate the problem without paying attention to the solution technique. If we omit the dummy vehicle from the original model formulation, and we let the model fix the appropriate y_i^h 's in the equation (20) without dummy vehicles, we come up with the same optimal solution. We delete the dummy vehicle to decrease the number of decision variables at hand.

The second reason for deleting the dummy vehicle out of this problem is related to nature of the problem. When we attempt to solve the starting point model without fixing y_i^h 's, we obtain a solution other than optimal solution. In other words, we let the model pick the appropriate y_i^h 's. It yields the following result: $x_{01}^1 = x_{10}^1 = 1$, and $x_{02}^2 = x_{23}^2 = x_{30}^2 = 1$, $z_1 = 150$, $z_2 = 50$, $z_3 = 150$ and $y_1^1 = y_2^2 = y_3^2 = 1$ with a total cost of 3515.3. Since the starting point model is mixed integer non-linear model, even *Hyper Lingo* cannot find the real optimal solution. It gives one of the local results.

After we obtained the same results, we decided to extend the model in different ways. We do focus on modeling.

The first milestone in the thesis process is to show whether it is possible to combine inventory/allocation, VRP and repair problems in a unique model. Regardless of the solution technique like Benders decomposition we focus on the formulating the

problem. The second milestone is the derivation of generalized Benders decomposition equations for this problem.

A General Look at the Possible Extensions from the Starting Model – New Decision Variables

After we determined that we have a possibility to solve the starting point problem without using Bender's decomposition method, we feel comfortable to deal with the starting point problem to extend in several directions. One of the possible extensions can be about focusing on the newsboy properties. The other one is related to the VRP side of the toy problem, especially focusing on the delivery part.

We should define the extra decision variables and parameters needed for extension of the problem.

$$x_{ij}^h = \begin{cases} 1 & \text{if } L.B \leq x_{ij}^{hup} \leq U.B. \text{ and/or } L.B \leq x_{ij}^{hdel} \leq U.B \\ 0 & \text{otherwise} \end{cases}$$

z_{ij} is a continuous variable that denotes the amount of allocation among available products of node i to node j .

$$(28) \quad z_{ij} = x_{ij}^{hup} + x_{ij}^{hdel}$$

z_{ii} is a continuous variable that denotes the amount of allocation among available products of node i to supply itself.

$$(29) \quad z_{ii} = x_{ii}^{hup} + x_{ii}^{hdel}$$

y_{ij}^h is a binary variable equal to one if vehicle h allocates the supply from node i to node j and equal to zero otherwise.

y_i is a binary variable equal to one if node i is such a sufficient node that it can deliver outside but not accept repaired items, equal to zero if node i is not sufficient itself so that it can accept supplies but not deliver outside.

t_i^h is the amount of time vehicle type h spends at demand point I (Chan, 1999 Draft:8-20).

$U(h)$ is the time that a vehicle h spends “on the road” (Chan, 1999 Draft:8-21).

Here are some examples for each new decision variables:

x_{ij}^h is the vehicle routing decision variable as described earlier, which denote vehicle h goes from node i to node j if it is equal to one; otherwise, zero. We add two component to the original vehicle routing decision variable such that these are related to what quantity we repaired at node i , and what quantity we hold in the inventory, x_{ij}^h up and x_{ij}^h del, respectively. The two new decision variables are lying within the upper bound (UB) and lower bound (LB).

We give two examples for each to understand what those are such that x_{23}^1 up denotes that vehicle 1 pick ups the repaired items at node 2 and delivers to node 3, x_{02}^2 up denotes that vehicle 2 pick ups the repaired item at node 0 and delivers to node 2. x_{23}^1 up is the decision variable based upon the repair capabilities of node 2, which is uniformly distributed between [0-9]. x_{02}^2 up is the decision variable based upon the repair capabilities of node 0, which is uniformly distributed between [0-15].

On the other hand, x_{23}^1 del denotes that vehicle 1 pick ups the inventoried items at node 2 and delivers to node 3, and x_{02}^2 del denotes that vehicle 2 pick ups the inventoried items at node 0 and delivers to node 2. x_{23}^1 del is the decision variable based upon the remaining inventory of node 2 at the end of the day, and x_{02}^2 del is the decision variable based upon the remaining inventory of node 0 at the end of the day.

The delivery variable in the toy problem, z_i , has two purpose. One of them is the amount of demand at node i , and the other one is the amount of delivery from main depot to node i . Actually, we have one index of z_i associated with one main depot and it is enough for the formulation of toy problem. When we assumed that every node can supply the other node and itself as well, then we add the second index to the delivery decision variable. z_{01} is the delivery amount from node zero to node one. This delivery amount is the sum of repaired and inventoried items that shipped from node zero to node one by any vehicle.

Since we are dealing with repaired items there should be some possibility to re-supply itself. We defined a new decision variable, z_{ii} , to show that each node can re-supply itself. z_{11} is the amount that node 1 re-supplied itself, and z_{22} is the amount that node 2 re-supplied itself. z_{11} is the sum of x_{1up} and x_{1del} . x_{1up} denotes that the repair capabilities are used for satisfying the local failures whereas x_{1del} denotes that we satisfy the failures by using the local inventory.

We can examine the placing order decision variable y^h_i in the same manner as we did to z_i . We add the third index to placing order constraints y^h_{ij} due to the fact that each node in the model is treated like a depot. y^1_{01} is equal to one if vehicle 1 allocates the supply from node zero to node one; otherwise zero. In other words, if $y^1_{01}=1$, then z_{01} can take value, otherwise it is zero.

The last new decision variable, y_i , is a binary variable that allows us to satisfy the assumption that either we can send an item to the other nodes or we can accept items. Since we are dealing with repaired items, it would make no sense during the period, say a day, send three items to outside, and accept one item. We add this variable to prevent

from happening this situation. If y_1 is equal to one, then it means that node 1 will be sufficient enough to re-supply itself and can supply the other nodes. If y_1 is equal to zero, then it means that node will use up all of its available resources, which are the amount of available inventory at hand, and available repair capabilities and accept some additional items from other nodes as well.

Thesis Problem

Since we are using the same distance matrix as we used in the toy problem, we do not need any update in the thesis problem. In addition to that we do not extend the toy problem in terms of VRP perspective in depth except we add crew duty time constraints including some constant loading and unloading time. In thesis problem, we keep the other assumptions of the VRP at the same level.

We mainly focused on the inventory/allocation perspective of the toy problem. Notice that, the main depot is the only resource to deliver to the other nodes in the toy problem whereas we set up the thesis problem in such a way that we can deliver from any nodes to any nodes with respect to the inventory cost function.

Table 5: The Inventory/Allocation Table for Thesis Problem

Node	0	1	2	3
Max. Inventory (D1)	10	4	5	5
Min. Inventory	0	0	0	0
Initial Inventory (β_i)	2	1	2	1
Demand (PDF)	0.1	0.25	0.4	0.2
Shortage cost (c_i)	\$5	\$20	\$25	\$15
Surplus cost (C_i)	\$10	\$5	\$5	%5
Newsboy cost function	$18 + 0.5z_{10} + 0.5z_{20} + 0.5z_{30}$	$48.75 - 3.75z_{01} - 3.75z_{21} - 3.75z_{31}$	$51 - 4z_{02} - 4z_{12} - 4z_{32}$	$39 - 2z_{03} - 2z_{13} - 2z_{23}$
Failure	-	[0-9]	[0-10]	[0-9]
Repair	[0-15]	[0-6]	[0-9]	[0-9]

The objective function

$$(30) \quad \text{Minimize } \sum_{i \in I} \sum_{j \in I} \sum_{h \in H} d_{ij} x_{ij}^h + \sum_i q_i(z_i)$$

The objective function of thesis problem, equation (29) is the same as the objective function of starting problem, which is equation (4). It is to minimize the inventory cost and traveling cost. It is noteworthy to explain the effect of our new assumption; that each node can supply the other nodes on the inventory cost function.

$$(31) \quad q_i(z_i) = \int_{\beta_i + z_{ij}}^{\infty} C_i(\xi - \beta_i - z_{ij}) dF_i(\xi) + \int_0^{\beta_i + z_{ij}} c(\beta_i + z_{ij} - \xi) dF_i(\xi), \quad i \in I - j, \quad \forall j$$

For example; the inventory cost function at node 1 is

$$(32) \quad q_1(z_1) = 48.75 - 3.75z_{01} - 3.75z_{21} - 3.75z_{31}$$

We will show how we get this inventory newsboy cost function for node one step by step.

Notice that the demand coefficients of node 1 are the same regardless to where it was supplied. You can see Table 4 for the information of the other nodes. As you recall, we can laterally supply from any node to any node in our model. It is clear that we use uniformly distributed demand for each node.

$$\int_{\beta_1+z_{01}}^{D1} c1 \frac{1}{D1} d\varepsilon + \int_0^{\beta_1+z_{01}} C1 \frac{1}{D1} d\varepsilon = \frac{65}{4} - \frac{15}{4} z_{01}$$

$$\int_{\beta_1+z_{21}}^{D1} c1 \frac{1}{D1} d\varepsilon + \int_0^{\beta_1+z_{21}} C1 \frac{1}{D1} d\varepsilon = \frac{65}{4} - \frac{15}{4} z_{21}$$

$$\int_{\beta_1+z_{31}}^{D1} c1 \frac{1}{D1} d\varepsilon + \int_0^{\beta_1+z_{31}} C1 \frac{1}{D1} d\varepsilon = \frac{65}{4} - \frac{15}{4} z_{31}$$

Node one can demand the same amount from every other node. When we sum up three demand functions, we come up with equation (32). We can derive the inventory cost functions for the remaining nodes in the model as shown below.

$$q_1(z_1) = 48.75 - 3.75z_{01} - 3.75z_{21} - 3.75z_{31}$$

$$q_2(z_2) = 51 - 4z_{02} - 4z_{12} - 4z_{32}$$

$$q_3(z_3) = 39 - 2z_{03} - 2z_{13} - 2z_{23}$$

$$q_0(z_0) = 18 + 0.5z_{10} + 0.5z_{20} + 0.5z_{30}$$

The demand function of node zero is positive because its shortage cost is less than its surplus cost. It is not a non-decreasing function.

The constraints are as follows:

Equation (7), equation (8), equation (9) and equation (10) are unchanged. We added complete set of sub-tour breaking constraints, equation (10), to the thesis problem just to be safe.

When we consider allocation/inventory constraints we put a lot of effort to modify into the new settings.

$$(33) \quad \sum_{i \in I} \sum_{\substack{j \in I \\ i \neq j}} z_{ij} y_{ij}^h \leq V(h) \quad h \in H$$

$$(34) \quad \sum_{j \in I} z_{ij} \leq \bar{P}_i \quad \forall i \in I$$

$$(35) \quad \sum_{i \in I} z_{ij} < Capacity_j \quad \forall j$$

$$(36) \quad \sum_{j \in M_j} x_{ij}^h - y_{ij}^h \geq 0 \quad \forall i, h \in H$$

$$(37) \quad \sum_{h \in H} \sum_{\substack{i \in I \\ i \neq j}} y_{ij}^h \leq 1 \quad \forall j$$

The equivalent of equation (12) is equation (33) in the thesis problem. It ensures that load capacity assigned to each vehicle h is within its capacity. Note that we can assign load to vehicle h from node i to node j if y_{ij}^h is equal to one; otherwise we can not.

The equivalent of equation (13) is equation (34) in the thesis problem. Since we have one main supplier in the starting point problem, we had one constraint for main depot. The number of depots in the thesis model will be as many as the number of nodes in the system. Therefore, we would have four supply constraints for each demand point. The number of units available plus the repair capability sum up to the supply quantity of each node. Node one has one unit at the end of the period, and it can repair most 6 units.

Node one can then supply at most seven units to the other nodes, including re-supplying itself as well.

There is a demand constraint for each demand point besides the supply constraints. Equation (35) ensures that demand cannot be exceeded at each demand point. The initial level of node one is 100 units and the maximum inventory level of node 1 is 500 units in the initial model. This means node one demand is at most 400 units. In the thesis model, the computation of demand for each demand point will be different for sure due to the fact that each node can re-supply itself. If the number of failures at each node minus available local resources, which the sum of inventoried items and repaired items is greater than zero, then we can send such a quantity that we satisfy the failures and maximize the inventory level, otherwise we can only maximize the inventory level associated with that node. For example, failures can be between zero and nine at node one. The initial level and maximum inventory level of node one is one, and four, respectively. The repair capability is up to six units. Failures, which is nine minus one at hand plus six units at repair equals to two. We have two units shortages, but node one can demand up to six units which is four units due to depot capacity plus two units shortages. We can calculate the demands of other nodes in the same fashion.

It is time to talk about the new additional constraints to satisfy our assumptions.

$$(38) \quad x_{ij}^h up + x_{ij}^h del - Mx_{ij}^h \leq 0$$

$$(39) \quad z_{ij} = \begin{cases} x_i up + x_i del & \text{if } i = j \\ x_{ij}^h up + x_{ij}^h del & \text{if } i \neq j \end{cases}$$

$$\begin{aligned}
& z_{ij} + My_j \geq 0 \quad (\text{i}) \\
& z_{ij} - My_j \leq 0 \quad (\text{ii}) \\
(40) \quad & z_{ij} + My_j \leq M \quad (\text{iii}) \\
& z_{ij} - My_j \geq -M \quad (\text{iv})
\end{aligned}$$

$$\begin{aligned}
(41) \quad & z_{jj} - \text{Local Resource}_i + My_j \geq 0 \\
& z_{jj} - My_j \geq -M
\end{aligned}$$

$$(42) \quad x_{iup} + x_{ij}^h up \leq R_i$$

$$(43) \quad x_{idel} + x_{ij}^h del \leq \beta_i$$

$$(44) \quad \sum_{i \in I} t_i^h \sum_{j \in I} x_{ij}^h + \sum_{i \in I} \sum_{j \in I} d_{ij}^h x_{ij}^h \leq V(h) \quad \forall h$$

$$(45) \quad z_{ij} - M \sum_{h \in H} y_{ij}^h \leq 0 \quad \forall i, j \in I$$

where big-M is the largest number that satisfies the related constraints.

Equation (38) assumes that we can deliver if the vehicle routing arc is open. We put all z 's in one place in equation (39) so that we can understand the relation between them.

Equation (40) and (41) are connected with each other. While the first one satisfies the condition that we either deliver the items to the other nodes or get the items out of the other nodes, the second one ensures that if we get the items out of other nodes we make sure that that node has used up all its available resources. I would like to give an example for these two constraints. Think about these two sets of constraints:

$$\begin{aligned}
(46) \quad & z_{01} + z_{21} + z_{31} + 50y_1 \Rightarrow 0 \quad (\text{i}) \\
& z_{10} + z_{12} + z_{13} - 50y_1 \leq 0 \quad (\text{ii}) \\
& z_{01} + z_{21} + z_{31} + 50y_1 \leq 50 \quad (\text{iii})
\end{aligned}$$

$$z_{10} + z_{12} + z_{13} - 50y_1 \Rightarrow -50 \text{ (iv)}$$

$$(47) \quad z_{11} - 7 + 50y_1 \Rightarrow 0$$

$$z_{11} - 50y_1 \Rightarrow -50$$

where big-M is 50, $i = 0, 2, 3$ and $j = 1$, and 7 in equation (47) corresponds to possible local resources, the sum of inventory at hand and repair capability, at node one. If y_1 is equal to zero, then equation (i) and (iii) will hold which means we can get the items out of other nodes, equation (ii) and (iv) will be relaxed which means we cannot deliver to the other nodes. If y_1 equal to one, then equation (ii) and (iv) holds, which means we can deliver to the other nodes, and equation (i) and (iii) will be relaxed, which means we cannot get the items out of the other nodes. Equation (44) satisfies that if y_1 is equal to zero, it forces to use all available resource, which are 7 in this case.

Equation (42) is the constraint related to repair capacity in the node. This repair capacity includes what it repairs for itself and for the other nodes.

Equation (43) is the constraint related to available inventory at each node. This constraint has the same the logic as the repair capacity. It includes what it supplies to itself and to the others out of depot inventory.

We modified the VRP constraints by adding equation (42), which satisfies the condition that we cannot exceed available crew-duty hours associated with each crew team. We assumed that loading and unloading takes a constant time such as half an hour in the model.

We accomplish the first milestone that is corresponding to formulating the thesis model. In the second milestone, we simplified our thesis problem so that we can apply generalized Benders' decomposition technique.

The Second Milestone

The second milestone includes the application of generalized Benders' decomposition technique to the thesis problem. We will do some simplification to the thesis problem so that we can apply the technique. We will keep the lateral supply attribute of the thesis problem in the second milestone. Every node can supply to every other node. We omit the some constraints out of thesis problem such as having "accept or deliver" option of each node, and tracking the inventoried and repaired items individually.

The challenging part of the generalized Benders' decomposition technique is to define the relaxed master problem, which is equivalent to the original problem. The Benders' cut for the starting point model is as follows:

$$(48) \quad z \geq \Omega \bar{P} + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-0} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-0} (\kappa_i^h + \varphi_i^h) y_i^h.$$

The first two summations are referring to the intercept of the Benders' cut, and the last two summations are representing the slope of the Benders cut. Since we have four depots in the second milestone, we should write down four additional cuts. The only difference from the starting model is adding more indexes to the cut.

The first cut is generated based upon main depot zero, which is shown in equation (49). The other cuts are generated based upon node one, two and three in equation (50), (51), and (52), respectively.

$$(49) \quad z \geq \Omega_0 \bar{P}_0 + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-0} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-0} (\kappa_i^h + \varphi_i^h) y_i^h.$$

$$(50) \quad z \geq \Omega_1 \bar{P}_1 + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-1} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-1} (\kappa_i^h + \varphi_i^h) y_i^h.$$

$$(51) \quad z \geq \Omega_2 \bar{P}_2 + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-2} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-2} (\kappa_i^h + \varphi_i^h) y_i^h.$$

$$(52) \quad z \geq \Omega_3 \bar{P}_3 + \sum_{h \in H} (\rho^h V_h + \kappa^h + \kappa_0^h) + \sum_{i \in I-3} \varphi_i^0 y_i^0 + \sum_{h \in H} \sum_{i \in I-3} (\kappa_i^h + \varphi_i^h) y_i^h.$$

We want to explain the inventory cost portion of the Benders' cut. In equation (49), the slope portion of the equation is based upon the operating (κ_i^h), and inventory cost (φ_i^h).

We explain the operating cost in the generalized Benders' section of Chapter Three explicitly. We modified the inventory cost function to the modified thesis problem. We can define the inventory cost functions associated with the first section of the equation (49) as follows.

$$(53) \quad \begin{aligned} \varphi_1^0 &= 48.75 - 3.75 \times (z_{01} + z_{21} + z_{31}) - (\Omega_0 - \rho_0) z_{01} - (\Omega_0 - \rho_0) z_{21} - (\Omega_0 - \rho_0) z_{31} \\ \varphi_2^0 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_0 - \rho_0) z_{02} - (\Omega_0 - \rho_0) z_{12} - (\Omega_0 - \rho_0) z_{32} \\ \varphi_3^0 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_0 - \rho_0) z_{03} - (\Omega_0 - \rho_0) z_{13} - (\Omega_0 - \rho_0) z_{23} \end{aligned}$$

The second portion of slope equation can be defined as follows.

$$\begin{aligned}
\varphi_1^0 &= 48.75 - 3.75 \times (z_{01} + z_{21} + z_{31}) - (\Omega_0 - \rho_0)z_{01} - (\Omega_0 - \rho_0)z_{21} - (\Omega_0 - \rho_0)z_{31} \\
\varphi_2^0 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_0 - \rho_0)z_{02} - (\Omega_0 - \rho_0)z_{12} - (\Omega_0 - \rho_0)z_{32} \\
\varphi_3^0 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_0 - \rho_0)z_{03} - (\Omega_0 - \rho_0)z_{13} - (\Omega_0 - \rho_0)z_{23} \\
\\
\varphi_1^1 &= 48.75 - 3.75 \times (z_{01} + z_{21} + z_{31}) - (\Omega_0 - \rho_1)z_{01} - (\Omega_0 - \rho_1)z_{21} - (\Omega_0 - \rho_1)z_{31} \\
(54) \quad \varphi_2^1 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_0 - \rho_1)z_{02} - (\Omega_0 - \rho_1)z_{12} - (\Omega_0 - \rho_1)z_{32} \\
\varphi_3^1 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_0 - \rho_1)z_{03} - (\Omega_0 - \rho_1)z_{13} - (\Omega_0 - \rho_1)z_{23} \\
\\
\varphi_1^2 &= 48.75 - 3.75 \times (z_{01} + z_{21} + z_{31}) - (\Omega_0 - \rho_2)z_{01} - (\Omega_0 - \rho_2)z_{21} - (\Omega_0 - \rho_2)z_{31} \\
\varphi_2^2 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_0 - \rho_2)z_{02} - (\Omega_0 - \rho_2)z_{12} - (\Omega_0 - \rho_2)z_{32} \\
\varphi_3^2 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_0 - \rho_2)z_{03} - (\Omega_0 - \rho_2)z_{13} - (\Omega_0 - \rho_2)z_{23}
\end{aligned}$$

We can generate Benders' cut for the node one in the same fashion as follows.

The first portion of slope in equation (50) is

$$\begin{aligned}
\varphi_0^0 &= 18 + 0.5 \times (z_{10} + z_{20} + z_{30}) - (\Omega_1 - \rho_0)z_{10} - (\Omega_1 - \rho_0)z_{20} - (\Omega_1 - \rho_0)z_{30} \\
(55) \quad \varphi_2^0 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_1 - \rho_0)z_{02} - (\Omega_1 - \rho_0)z_{12} - (\Omega_1 - \rho_0)z_{32} \\
\varphi_3^0 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_1 - \rho_0)z_{03} - (\Omega_1 - \rho_0)z_{13} - (\Omega_1 - \rho_0)z_{23}
\end{aligned}$$

and the second portion of slope in equation (50) is

$$\begin{aligned}
\varphi_0^0 &= 18 + 0.5 \times (z_{10} + z_{20} + z_{30}) - (\Omega_1 - \rho_0)z_{10} - (\Omega_1 - \rho_0)z_{20} - (\Omega_1 - \rho_0)z_{30} \\
\varphi_2^0 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_1 - \rho_0)z_{02} - (\Omega_1 - \rho_0)z_{12} - (\Omega_1 - \rho_0)z_{32} \\
\varphi_3^0 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_1 - \rho_0)z_{03} - (\Omega_1 - \rho_0)z_{13} - (\Omega_1 - \rho_0)z_{23} \\
\\
\varphi_0^1 &= 18 + 0.5 \times (z_{10} + z_{20} + z_{30}) - (\Omega_1 - \rho_1)z_{10} - (\Omega_1 - \rho_1)z_{20} - (\Omega_1 - \rho_1)z_{30} \\
(56) \quad \varphi_2^1 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_1 - \rho_1)z_{02} - (\Omega_1 - \rho_1)z_{12} - (\Omega_1 - \rho_1)z_{32} \\
\varphi_3^1 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_1 - \rho_1)z_{03} - (\Omega_1 - \rho_1)z_{13} - (\Omega_1 - \rho_1)z_{23} \\
\\
\varphi_0^2 &= 18 + 0.5 \times (z_{10} + z_{20} + z_{30}) - (\Omega_1 - \rho_2)z_{10} - (\Omega_1 - \rho_2)z_{20} - (\Omega_1 - \rho_2)z_{30} \\
\varphi_2^2 &= 51 - 4 \times (z_{02} + z_{12} + z_{32}) - (\Omega_1 - \rho_2)z_{02} - (\Omega_1 - \rho_2)z_{12} - (\Omega_1 - \rho_2)z_{32} \\
\varphi_3^2 &= 39 - 2 \times (z_{03} + z_{13} + z_{23}) - (\Omega_1 - \rho_2)z_{03} - (\Omega_1 - \rho_2)z_{13} - (\Omega_1 - \rho_2)z_{23}
\end{aligned}$$

If we generate a Benders' cut based upon node zero, we show that we deliver to node one, two and three explicitly in equation (54). If we generate a Benders' cut based upon node one, we show that we deliver to node zero, two and three explicitly in equation (56). We can generate the cuts based upon the other nodes as well. Notice that the intercept of each cut regardless of supply node is the same. Since we ran out of time, we could not provide the numerical results.

Now we are ready to talk about the solution strategies we come up with to solve the thesis model.

Chapter 4 - Analysis

Introduction

We showed the formulation of thesis problem in the methodology section. We will show the results of a thesis problem without using Generalized Benders decomposition technique that corresponds to the result of first milestone in the thesis process.

First Milestone

We would like to give you some information about our original model that we show the mathematical formulation of it at Appendix 10. We have a total of 124 variables and 117 constraints in the model. 52 of them are binary, and the rest of them are continues. Binary variables are placing order variables (y_{ij}^h), vehicle routing variables (x_{ij}^h) and switch on-off variables related to either sending or receiving items (y_i). We have only two non-linear constraints in the model associated with vehicle capacity constraints that correspond to the 90th and 91st constraints. We have four nodes, and two vehicles.

If we do not fix the y_{ij}^h 's, the branch-and-bound can procedure can take 2^{52} solution trees. Each time the software Lingo can follow the different path due to the different storage memory produced to solve the given problem (Lingo Technical assistance). The result of the first milestone can be summarized as follows.

Table 6 : The result of the first milestone

		First solution	Second solution (Alternative)
Objective value		$Z^* = 133.55$	$Z^* = 133.55$
Routes		$x_{02}^1 - x_{23}^1 - x_{30}^1$ $x_{01}^2 - x_{10}^2$	$x_{03}^1 - x_{32}^1 - x_{20}^1$ $x_{01}^2 - x_{10}^2$
	Initial position	-	
Node 0	Initial level:2 Max.level:10 Repair Cap:[0-15]	$Z_{00} = 7$ ($x_{0up}=7$)	$Z_{00} = 11$ ($x_{0del} =2, x_{0up}=9$)
Node 1	Initial level:1 Max.level:4 Repair Cap.: [0-6] Failure : [0-9]	$Z_{11} = 7$ ($x_{1del} =1, x_{1up}=6$) $Z_{01}=6$ ($x_{01up}^2 = 4, x_{01del}^2=2$)	$Z_{11} = 7$ ($x_{1del} =1, x_{1up}=6$) $Z_{01}=6$ ($x_{01up}^2 = 6$)
Node 2	Initial level:2 Max.level:5 Repair Cap:[0-9] Failure : [0-10]	$Z_{22}= 11$ ($x_{2del} =2, x_{2up}=9$) $Z_{02} =4$ ($x_{02up}^1=4$)	$Z_{22}= 11$ ($x_{2del} =2, x_{2up}=9$) $Z_{32} =4$ ($x_{32up}^1=3, x_{32del}^1= 1$)
Node 3	Initial level:1 Max.level:5 Repair Cap:[0-9] Failure:[0-9]	$Z_{33} = 10$ ($x_{3del} =0.99999, x_{3up}=9$)	
		$y_{01}^2 = y_{02}^1 = y_{03}^1 =1$ $y_3 = y_0 = 1$	$y_{01}^2 = y_{32}^1 = y_{03}^1 =1$ $y_3 = y_0 = 1$

We can go backward to analyze the situation such that we send six serviceable parts to node 1. There can be at most nine failures at node one, but node one can meet seven of them by using our local resource. It means node 1 use all of his available repair and inventory to meet the failures. We are a shortage of two serviceable parts. Since the maximum inventory capacity is four, we can demand at most six serviceable parts to

stock up to the maximum level. Node one got four serviceable items out the repaired shop and two serviceable items out of the inventory out of node zero.

For node two, we can have at most ten failures during that period. But we can satisfy eleven of them by using the local resource. Node two uses all of its repair capabilities and inventory to satisfy the failures. Since our maximum inventory level is four, we can demand at most four serviceable items from any node in the system. Node two got four units out of the repaired shop of node zero.

For node three, we have nine failures but it meets all the failures by using its own local resource, besides that it has a surplus of one. Since y_3 is equal to one, it is ready to send it to the other nodes.

The placing order variables are consistent with the result. Since we are sending out of node zero, $y^2_{01} = y^1_{02} = y^1_{03}$ are equals to one. Besides that, we deliver to node one by vehicle two, therefore, placing an order constraint y^2_{01} and associated vehicle routing variables such as $x^2_{01} - x^2_{10}$ is equal to one. y^1_{02} is equal to one, because we are using vehicle one goes out of node two that is consistent with the constraint set. We do send nothing from node three to any other node even though y_3 is equal to one, because we satisfied the demand requirements of node one and two. We cannot deliver to node zero, because it supplies to node one. On the other hand, the switch on/off variables y_1 , and y_2 are equal to zero mean that node one and node two can only demand some send serviceable items from the other nodes. In this result, y_3 did send nothing because node one and two are satisfied by node zero. If node zero did not have enough to send them, then node three would be ready to satisfy their demands. Node three has only one surplus item to deliver to the other nodes.

Vehicle two goes to node one and come back. The total time spent of vehicle two is the sum of traveling time from node zero to node 1, plus 0.5 hour for unloading and plus traveling time from node one to node zero, which is equal to six and half hour. The that of vehicle one is sum of traveling time from node zero to node two, plus unloading time at node two, plus traveling time from node two to three, and traveling time from node three to node zero, which is equal to nine hours and eight minutes. Note that vehicle two delivers nothing to node three because node three is a candidate to deliver to the other nodes due to its low shortage cost compare to node one and two.

We have an alternative solution to this problem. We have the same objective function with a value of 133.55, but we satisfy the demands of node one and two from node zero and node three.

For node one, we are sending six repaired items out of node zero to node one to satisfy the demand. This case node three delivers three repaired and one inventoried item to node two to satisfy the demand of node two. Node one and two have the same number of failures as before such that they got the same number of serviceable items. But there is a big difference. At this time, node three can be able to send six more units to the node two if there is a need.

The switch on/off constraints for delivery, y_0 and y_3 , are consistent with the results. Remember y_0 is equal to one means that we can deliver out of node zero. y_3 is equal to one means that we can deliver out of node three.

The placing order constraints, $y^2_{01}-y^1_{32}-y^1_{03}$ are consistent with the result. Since vehicle two goes out of node two, y^2_{01} arc is open. By the same idea, vehicle one visits

node three and two. The other placing order constraints are consistent with the result, as well. See the complete list of non-zero decision variables at Appendix 11.

We may interpret these two results as follows. In the first run, we satisfy the demand out of node zero. It means node zero it is the supply source for the model in order to get the optimal result. In the second run, the supply source is the node three and node zero. In this problem instance we have two options to satisfy the demands. We may think of it in terms of changing the place of supply sources. This happens usually in the deployment in military..

With the current newsboy inventory costs, we may think that we may deploy all the aircraft from node three to node one and/or two and treat the node three as secondary main depot.

What would happen if we change the demand quantities of nodes one, two, or three. We want to test the model whether it has a debug or not. The new set of demand quantities is such that node one requires four serviceable items in stead of six, node two requires the same amount, that is four serviceable items, and node three requires six serviceable items in stead of four. The demand constraints for nodes one, two, and three are the 87th, 88th, and 89th ones at Appendix 10.

We displayed the results in Table 2. We would like to talk about only the first run. Since we changed the demand quantity of node one from six to four, we can supply up to four units. Node one takes only a total of four units out of node zero. These are 3.07 repaired items and 0.92 inventoried item. Note that we defined these variables as continuous. Node two takes a total of four serviceable items out of node three such that one of them is from inventory and the rest is from repaired shop.

Table 7: The Demands of Node one and three Modified

		First solution	Second solution (Alternative)
Objective value		$Z^* = 141.05$	$Z^* = 141.05$
Routes		$X_{03}^2 - X_{32}^2 - X_{20}^2$ $X_{01}^1 - X_{10}^1$	$X_{02}^1 - X_{23}^1 - X_{30}^1$ $X_{01}^2 - X_{10}^2$
	Initial position	-	-
Node 0	Initial level:2 Max.level:10 Repair Cap:[0-15]	$Z_{00} = 13$ ($x_{0up}=11.93, x_{0del}=1.07$)	$Z_{00} = 9$ ($x_{0del} =2, x_{0up}=7$)
Node 1	Initial level:1 Max.level:4 Repair Cap.: [0-6] Failure : [0-9]	$Z_{11} = 7$ ($x_{1del} =1, x_{1up}=6$) $Z_{01}=4$ ($x_{01up} = 3.07, x_{01del}=0.93$)	$Z_{11} = 7$ ($x_{1del} =1, x_{1up}=6$) $Z_{01}=4$ ($x_{01up} = 6$)
Node 2	Initial level:2 Max.level:5 Repair Cap:[0-9] Failure : [0-10]	$Z_{22}= 11$ ($x_{2del} =2, x_{2up}=9$) $Z_{32} =4$ ($x_{02up}=3, x_{02del}=1$)	$Z_{22}= 11$ ($x_{2del} =2, x_{2up}=9$) $Z_{02} =4$ ($x_{32up}=3, x_{32del}= 1$)
Node 3	Initial level:1 Max.level:5 Repair Cap:[0-9] Failure:[0-9]	-	$Z_{33} = 10$ ($x_{3del} =0.99999, x_{3up}=9$)
		$y_{01}^1 = y_{32}^2$ $y_3 = y_0 = 1$	$y_{01}^2 = y_{02}^1$ $y_3 = y_0 = 1$

Note that y_{01}^1, y_{32}^2 are consistent with the routes and delivery variables. Vehicle one visits and delivers to node one, vehicle two visits and delivers to node two. y_3, y_0 are consistent with the results as well. We deliver out of node three and node zero.

We generate three solutions to illustrate the importance of the combining the inventory and vehicle routing problems.

Table 8 : The result of the thesis problem

options	z_i	Operating cost	Inventory cost	Total cost
1	$Z_{03} = 4$ $Z_{21} = 6$	14.5	126.25	140.75
2	$Z_{01} = 6$ $Z_{02} = 4$	17	118.25	135.25
3	$Z_{01} = 6$ $Z_{32} = 4$	15.3	118.25	133.55

In the first iteration, we follow the shortest route to satisfy the demands. If we want to satisfy the demands in the quickest way, we will apply the first option. Notice that the total cost is the highest among the possible options for this numerical example. The second and third option has the same inventory cost function, but different operating costs. Notice that in the second and third option, the demand points get what they need. Node two gets 4 units out of node zero in the first option, or out of node three in the second option. The third option results in a lowest cost. Even though we are satisfying the customer demands a little bit slower than we do in the first option, we have the optimal results. This table shows us that we will have a lower cost if we consider inventory/allocation and vehicle routing together. Even though we extend Federgruen and Zipkin in the thesis model, we come up with the same conclusion as they found in their study.

The impact of changing the number of nodes and vehicles on the problem

The impact of adding one more node into model can be summarized as follows:

It is for sure that we expand the number of constraints while adding the number of nodes into the model. We try to mention the impact of it on the each type of constraint. We add one more constraint for out/in of main depot constraints, which are equation (7) and (8), respectively. We add one for each vehicle type for the route continuity constraint, which corresponds to equation (9). Since we are not changing the number of vehicles in the model, we will have the same number of constraints in terms of tracking the vehicles on the way to main depot, which corresponds, to equation (10). We add four additional sub-tour breaking constraints for each vehicle type, which is originally equation (11). Since equation (33) is related to vehicle capacity constraints, it will have the same number of constraints. We add one more constraint for equation (34), and (35). We will add one placing order constraint for each vehicle type for equation (36). Since the equation (37) is based on the number of nodes, we will add one more constraint. The situation for equation (38) will be different. We add one for each previous node, and five for the new coming node and two for the equation (39). The number of equations for (40) will remain the same, but we add two more constraints for equation (41), (42), and (43). The equation (45) will be the same.

When we think of multi-item in our model, we add one more index to the delivery variables, and add a newsboy cost function for each item. The new delivery variable will be z_{ijk} , denotes item k goes from i to j . The common resources will be vehicle routing constraints, and vehicle capacity constraints. Every item can be analyzed independently.

Every possible solution can pass up to the common resources. As soon as we satisfy the common resources, we assumed that we reached the optimal solution.

We may think of adding more nodes in a generic way as well. We will present the equations again for the sake of completeness. We will assume that we add n nodes more, which can be denoted like $|I'|$.

$$(7) \quad \sum_{j \in I} \sum_{h \in H^r} x_{ij}^h = \begin{cases} |H^r| & \text{if } i = 0 \\ 1 & \text{if } i = 1, 2, \dots, |I| \end{cases}$$

$$(8) \quad \sum_{i \in I} \sum_{h \in H^r} x_{ij}^h = \begin{cases} |H^r| & \text{if } j = 0 \\ 1 & \text{if } j = 1, 2, \dots, |I| \end{cases}$$

Equation (7) and equation (8) are going to increase by $|I'|$. In other words the new number of constraints for equation (7) and (8) will be $|I| + |I'|$.

$$(9) \quad \sum_{i \in M_p} x_{ij}^h - \sum_{j \in M_p} x_{ij}^h = 0 \quad \forall h, \forall p \in I.$$

Equation (9) is going to increase by $|H^r|$ times $|I'|$. This means we write down the new set of constraints for each vehicle type.

$$(10) \quad \sum_{j \in M_0} x_{0j}^h \leq 1 \quad \forall h, \quad \text{and} \quad \sum_{i \in M_0} x_{i0}^h \leq 1 \quad \forall h$$

The number of constraints in equation (10) remains the same due to the fact that it depends on the vehicles.

$$(11) \quad \sum_{i \in L} \sum_{j \in L} x_{ij}^h \leq |L| - 1 \quad L \subseteq \{2, \dots, |I|\}, \quad 2 \leq |L| \leq |I| - 1; \quad h \in H$$

Constraint (11) is going to increase in the following fashion. $Cr(I, I-1)$ denotes the combination of "I chooses I-1". If we add I' nodes to the problem, then the number of constraints is the summation of $Cr(I+I', I+I')$, $Cr(I+I', I+I'-1)$, $Cr(I+I', I+I'-2)$, up to

$Cr(I+I', 1)$. Remember node zero is the main depot. In addition to that multiply each combination by the number of vehicles.

$$(33) \quad \sum_{i \in I} \sum_{\substack{j \in I \\ i \neq j}} z_{ij} y_{ij}^h \leq V(h) \quad h \in H$$

Equation (33) remains the same due to the fact that it based on the number of vehicle types.

$$(34) \quad \sum_{j \in I} z_{ij} < \bar{P}_i \quad \forall i \in I$$

Equation (34) is going to increase by $|I'|$. In other words the new number of constraints for equation (34) will be $|I| + |I'|$.

$$(35) \quad \sum_{i \in I} z_{ij} < capacity_j \quad \forall j$$

Equation (35) is going to increase by the $|I'|$.

$$(36) \quad \sum_{j \in \bar{M}_j} x_{ij}^h - y_{ij}^h \geq 0 \quad \forall i, h \in H$$

Equation (36) is going to increase by $|H''|$ times $|I'|$. In our case, if we add two more nodes, the additional number of constraints to the original would be four.

$$(37) \quad \sum_{h \in H} \sum_{\substack{i \in I \\ i \neq j}} y_{ij}^h \leq 1 \quad \forall j$$

Equation (37) is going to increase by $|I'|$.

$$(38) \quad x_{ij}^h up + x_{ij}^h del - Mx_{ij}^h \leq 0$$

Suppose that the total number of nodes would be equal to $|I'_{new}|$ after adding $|I'|$ nodes to the original problem. Then, we have the following number of constraints: $|I'_{new}| \cdot |I'_{new}| \cdot |H''|$.

$$(39) \quad z_{ij} = \begin{cases} x_{i,up} + x_{i,del} & \text{if } i = j \\ x_{ij}^h up + x_{ij}^h del & \text{if } i \neq j \end{cases}$$

We can define z_{ij} for two different situations. If $i=j$, then we will add $|\Pi'|$ additional nodes. If $i \neq j$, then $|\Pi'_{new}| - 1 \cdot |\Pi'_{new}|$ constraints are added.

$$(40) \quad \begin{aligned} z_{ij} + My_j &\geq 0 & \text{(i)} \\ z_{ij} - My_j &\leq 0 & \text{(ii)} \\ z_{ij} + My_j &\leq M & \text{(iii)} \\ z_{ij} - My_j &\geq -M & \text{(iv)} \end{aligned}$$

Equation (40) is composed of four sub constraints. It is going to increase by $4|\Pi'|$.

$$(41) \quad \begin{aligned} z_{ij} - \text{Local Resource}_i + My_j &\geq 0 \\ z_{ij} - My_j &\geq -50 \end{aligned}$$

Equation (42) is composed of two sub constraints. It is going to increase by $2|\Pi'|$.

$$(42) \quad x_{i,up} + x_{ij}^h up \leq R_i$$

$$(43) \quad x_{i,del} + x_{ij}^h del \leq \beta_i$$

Equation (42) and (43) are going to increase by $|\Pi'|$.

$$(44) \quad \sum_{i \in I} t_i^h \sum_{j \in I} x_{ij}^h + \sum_{i \in I} \sum_{j \in I} d_{ij}^h x_{ij}^h \leq V(h) \quad \forall h$$

Equation (44) is based upon the number of vehicles.

$$(45) \quad z_{ij} - M \sum_{h \in H} y_{ij}^h \leq 0 \quad \forall i, j \in I$$

The new number of constraints for equation (45) would be $|\Pi'_{new}| - 1 \cdot |\Pi'_{new}|$.

We can answer the impact of the adding more vehicles to the problem as well.

$|V'|$ denotes additional vehicles added into the original model.

Equation (9), (10), (33), (36), and (44) are going to increase by $|V'|$. Equation (11) is going to increase by the multiplication of $|V'|$ with every combination.

Chapter 5

Summary of This Research

The thesis process started with the toy problem that we studied throughout the process. Its size was small enough to capture the initial requirements of the thesis problem. It was complex enough to be a good candidate for thesis problem.

Firstly, we solved and understand the toy problem and its environment. It was composed of inventory and vehicle routing problems. After we felt comfortable on the toy problem, we decided to head for the formulating the thesis problem.

We wanted to add the third component, which is repair to the toy problem. It was a challenging task to combine all three components in a unique model. The first question that we wanted to answer if it is doable or to what extend it is doable.

We draw the boundary of the problems as follows. There is a repair capability for each base other than the main depot as well. There is a newsboy inventory-cost function associated with each base to the every other base. We set up a model for repairable items only. Each base has two options: accept the repairable items, or deliver the repairable items, but not both. If the base chooses the “accept” option, we make sure that that base has used all its available resources, initial inventoried items and repair capabilities, before it receives re-supplies. In the transportation/delivery submodel, we place limitation on the crew duty hours available.

The objective function is to satisfy the demands by minimizing traveling cost and inventory cost. Satisfying the demands is more dominant than minimizing traveling cost; therefore, the satisfying the demand is the driving factor in the objective.

Conclusion

The purpose of this model can be summarized in three steps. In the first step, we analyze and comprehend the inventory/allocation model (Federgruen and Zipkin, 1984:1019). Here we take an advantage of a numerical example in Chan's textbook (Chan, 1999-Draft: 9-6). In the second step, we would apply the generalized Benders decomposition technique to the problem as Federgruen and Zipkin did. Benders cut is playing a major role in the solution process. Since the Benders cut is composed of dual variables, we performed a sensitivity analysis for the dual variables associated with main depot capacity and vehicle capacity constraints. The starting point model shows us that combining vehicle routing and inventory saves a lot of resource in terms of time or money. In the third step, we extend the starting point model, which corresponds to our thesis model, in such a way that we let the model issue the lateral supply between every nodes in the model. We track the items individually; namely, they may come from either inventory or repaired shop of the node. When we examined the result of our thesis problem, we saw that we would have some savings in terms of time and money if we combined the inventory and vehicle routing together as Federgruen and Zipkin did in their study. Since we ran out of time we could not show the result of the generalized Benders' decomposition equations for the second milestone. But it is not working properly.

Suggestions and Recommendations

We came up with the mixed integer programming since we used the uniformly distributed demand function.

Since I run this model a couple of times, running an experiment on the parameters of the thesis problem may yield interesting results. The computational time can be a huge hindrance if you expand the nodes in the model. Therefore, it would be good to apply the Generalized Benders decomposition technique to thesis problem so that you can run the model as much as you want in a short computational time.

The second advantage of Generalized Benders decomposition is to have an opportunity to be able to attack Exponential distributed demands at inventory problems and Poisson distributed demands at repair problems.

This extension is valid for Turkish Air Force such that it would be useful to include multi-item in stead of expanding the model in terms of number of nodes. At most 20-25 items will be enough for Turkish Air Force.

Another extension can be the use of heterogeneous vehicles in the model. Since Turkish Air Force has three different kinds of vehicles for transportation in the inventory, it would result in a very useful model. These three aircraft types are stationed at two different bases. Formulating the problem by assuming that those are stationed at two different bases will add a complexity to the problem.

The loading and unloading times can be a variable associated with a known distribution instead of constant values. This allows us to focus on the land operations side of vehicle routing problem.

Appendix 1: The TSP Sub-problem for Starting Model

$$\begin{aligned} \text{MIN} \quad & 3.8 X_{032} + 5 X_{031} + 2 X_{030} + 3.8 X_{023} + 4 X_{021} + 3.5 X_{020} + 5 X_{013} \\ & + 4 X_{012} + 3 X_{010} + 2 X_{003} + 3.5 X_{002} + 3 X_{001} + 3.8 X_{232} \\ & + 5 X_{231} + 2 X_{230} + 3.8 X_{223} + 4 X_{221} + 3.5 X_{220} + 5 X_{213} \\ & + 4 X_{212} + 3 X_{210} + 2 X_{203} + 3.5 X_{202} + 3 X_{201} + 3.8 X_{132} \\ & + 5 X_{131} + 2 X_{130} + 3.8 X_{123} + 4 X_{121} + 3.5 X_{120} + 5 X_{113} \\ & + 4 X_{112} + 3 X_{110} + 2 X_{103} + 3.5 X_{102} + 3 X_{101} \end{aligned}$$

SUBJECT TO

- 2) $X_{230} + X_{220} + X_{210} + X_{130} + X_{120} + X_{110} = 2$
- 3) $X_{031} + X_{021} + X_{001} + X_{231} + X_{221} + X_{201} + X_{131} + X_{121} + X_{101} = 1$
- 4) $X_{032} + X_{012} + X_{002} + X_{232} + X_{212} + X_{202} + X_{132} + X_{112} + X_{102} = 1$
- 5) $X_{023} + X_{013} + X_{003} + X_{223} + X_{213} + X_{203} + X_{123} + X_{113} + X_{103} = 1$
- 6) $X_{203} + X_{202} + X_{201} + X_{103} + X_{102} + X_{101} = 2$
- 7) $X_{013} + X_{012} + X_{010} + X_{213} + X_{212} + X_{210} + X_{113} + X_{112} + X_{110} = 1$
- 8) $X_{023} + X_{021} + X_{020} + X_{223} + X_{221} + X_{220} + X_{123} + X_{121} + X_{120} = 1$
- 9) $X_{032} + X_{031} + X_{030} + X_{232} + X_{231} + X_{230} + X_{132} + X_{131} + X_{130} = 1$
- 10) $X_{131} + X_{121} - X_{113} - X_{112} - X_{110} + X_{101} = 0$
- 11) $X_{231} + X_{221} - X_{213} - X_{212} - X_{210} + X_{201} = 0$
- 12) $X_{031} + X_{021} - X_{013} - X_{012} - X_{010} + X_{001} = 0$
- 13) $X_{132} - X_{123} - X_{121} - X_{120} + X_{112} + X_{102} = 0$
- 14) $X_{232} - X_{223} - X_{221} - X_{220} + X_{212} + X_{202} = 0$
- 15) $X_{032} - X_{023} - X_{021} - X_{020} + X_{012} + X_{002} = 0$
- 16) $X_{132} - X_{131} - X_{130} + X_{123} + X_{113} + X_{103} = 0$
- 17) $X_{232} - X_{231} - X_{230} + X_{223} + X_{213} + X_{203} = 0$
- 18) $X_{032} - X_{031} - X_{030} + X_{023} + X_{013} + X_{003} = 0$
- 19) $Y_{11} + X_{113} + X_{112} + X_{110} \geq 0$
- 20) $Y_{21} + X_{213} + X_{212} + X_{210} \geq 0$
- 21) $X_{013} + X_{012} + X_{010} \geq 1$
- 22) $Y_{12} + X_{123} + X_{121} + X_{120} \geq 0$
- 23) $X_{223} + X_{221} + X_{220} \geq 1$
- 24) $Y_{02} + X_{023} + X_{021} + X_{020} \geq 0$
- 25) $X_{132} + X_{131} + X_{130} \geq 1$
- 26) $Y_{23} + X_{232} + X_{231} + X_{230} \geq 0$
- 27) $Y_{03} + X_{032} + X_{031} + X_{030} \geq 0$
- 28) $X_{132} + X_{131} + X_{123} + X_{121} + X_{113} + X_{112} \leq 2$
- 29) $X_{232} + X_{231} + X_{223} + X_{221} + X_{213} + X_{212} \leq 2$
- 30) $X_{032} + X_{031} + X_{023} + X_{021} + X_{013} + X_{012} \leq 2$

END

INTE Y11

INTE Y21

INTE Y01

INTE Y12

INTE Y32

INTE Y02
INTE Y13
INTE Y23
INTE Y03

Appendix 2: The Solution to TSP Sub-problem for the Starting Model

Global optimal solution found at step: 24
Objective value: 17.00000
Branch count: 0

Variable	Value	Reduced Cost
X103	1.000000	0.000000
X130	1.000000	0.000000
X202	1.000000	0.000000
X220	1.000000	0.000000
X001	1.000000	0.000000
X010	1.000000	0.000000
Y01	1.000000	0.000000
Y22	1.000000	0.000000
Y13	1.000000	0.000000

Appendix 3: The Inventory/Allocation Sub-problem for Starting Model

MIN - 6.5 Z3 - 7.5 Z2 - 8 Z1

SUBJECT TO

2) $Z3 + Z2 + Z1 \leq 400$

3) $Z1 \leq 400$

4) $Z2 \leq 50$

5) $Z3 \leq 350$

6) $Z1 \leq 0$

7) $Z3 \leq 150$

8) $Z2 \leq 200$

END

Appendix 4: The Solution to the The Inventory/Allocation Sub-problem for Starting

Model

Global optimal solution found at step: 3

Objective value: -1350.000

Variable	Value	Reduced Cost
Z2	50.00000	0.0000000
Z3	150.0000	0.0000000

Row	Slack or Surplus	Dual Price
4	0.0000000	7.500000
6	0.0000000	8.000000
7	0.0000000	6.500000

Appendix 5: The Calculation of Master Cut for the Master Problem (Starting)

ITERATION 1

$$\Omega := 0 \quad z1 := 0 \quad z2 := 50 \quad z3 := 150$$

$$\rho := \begin{bmatrix} 8 \\ 6.5 \\ 0 \end{bmatrix} \quad \kappa := \begin{bmatrix} 6 \\ 4 \\ 7 \end{bmatrix} \quad \kappa h0 := \begin{bmatrix} 3 \\ 2 \\ 3.5 \end{bmatrix} \quad \kappa hi := \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 3.5 \\ 0 & 2 & 0 \end{bmatrix} \quad z := \begin{bmatrix} z1 \\ z2 \\ z3 \end{bmatrix} \quad v := \begin{bmatrix} 0 \\ 150 \\ 200 \end{bmatrix}$$

$$v := \begin{bmatrix} 2600 - 8 \cdot z1 - (\Omega + \rho_1) \cdot z1 & 2600 - 8 \cdot z1 - (\Omega + \rho_2) \cdot z1 & 2600 - 8 \cdot z1 - (\Omega + \rho_3) \cdot z1 \\ 437.5 - 7.5 \cdot z2 - (\Omega + \rho_1) \cdot z2 & 437.5 - 7.5 \cdot z2 - (\Omega + \rho_2) \cdot z2 & 437.5 - 7.5 \cdot z2 - (\Omega + \rho_3) \cdot z2 \\ 3012.5 - 6.5 \cdot z3 - (\Omega + \rho_1) \cdot z3 & 3012.5 - 6.5 \cdot z3 - (\Omega + \rho_2) \cdot z3 & 3012.5 - 6.5 \cdot z3 - (\Omega + \rho_3) \cdot z3 \end{bmatrix}$$

$$\text{sum1} := -(v + \kappa hi)$$

$$v0 := \begin{bmatrix} 2600 - 8 \cdot z1 - (\Omega + \rho_1) \cdot z1 \\ 437.5 - 7.5 \cdot z2 - (\Omega + \rho_1) \cdot z2 \\ 3012.5 - 6.5 \cdot z3 - (\Omega + \rho_1) \cdot z3 \end{bmatrix} \quad v0 = \begin{bmatrix} 2.6 \cdot 10^3 \\ -337.5 \\ 837.5 \end{bmatrix}$$

$$v = \begin{bmatrix} 2.6 \cdot 10^3 & 2.6 \cdot 10^3 & 2.6 \cdot 10^3 \\ -337.5 & -262.5 & 62.5 \\ 837.5 & 1.063 \cdot 10^3 & 2.038 \cdot 10^3 \end{bmatrix}$$

THE MATRIX v CORRESPONDS TO THE INVENTORY COST OCCURED AT EACH DEMAND POINT. EACH ROW CORRESPONDS TO THE NODE 1, 2, and 3, RESPECTIVELY. EACH COLUMN CORRESPONDS TO THE VEHICLE 0, 1, and 2 RESPECTIVELY.

$$(-v0)_{1,1} + (\text{sum1})_{1,1} = -5.203 \cdot 10^3$$

$$(-v0)_{2,1} + \text{sum1}_{2,1} = 675$$

$$(-v0)_{3,1} + \text{sum1}_{3,1} = -1.675 \cdot 10^3$$

THESE ARE THE COEF. OF DUMMY VEHICLE PLACING ORDER VARIABLES ($y01$ - $y02$ - $y03$) IN THE MASTER CUT.

$$(\text{sum1}) = \begin{bmatrix} -2.603 \cdot 10^3 & -2.6 \cdot 10^3 & -2.6 \cdot 10^3 \\ 337.5 & 262.5 & -66 \\ -837.5 & -1.065 \cdot 10^3 & -2.038 \cdot 10^3 \end{bmatrix}$$

THE SECOND AND THIRD COLUMN CORRESPOND TO THE VEHICLE 1, AND 2. FOR EXAMPLE SECOND COLUMN, FIRST ROW IS THE COEF. OF $y11$, THIRD ROW IS THE COEF. OF $y13$ or THE SECOND COLUMN, FIRST ROW IS THE COEF. OF $y21$ IN THE MASTER PROBLEM.

$$\left[\sum_{h=1}^3 \kappa_h + \kappa h0_h + \rho_h \cdot v_h \right] \Rightarrow 1000.5$$

THIS IS THE RIGHT-HAND SIDE OF MASTER CUT.

Appendix 6: The Formulation of Master Problem for the Starting Model

```
MIN    Z
SUBJECT TO
2)- 5203 Y01 + 675 Y02 - 1675 Y03 - 2600 Y11 + 262.5 Y12 - 1065 Y13
   - 2600 Y21 - 66 Y22 - 2038 Y23 + Z >= 1000.5
3) 50 Y02 + 150 Y03 <= 0
4) 50 Y12 + 150 Y13 <= 150
5) 50 Y22 + 150 Y23 <= 200
6) Y01 + Y11 + Y21 = 1
7) Y02 + Y12 + Y22 = 1
8) Y03 + Y13 + Y23 = 1

END
INTE Y01
INTE Y02
INTE Y03
INTE Y11
INTE Y12
INTE Y13
INTE Y21
INTE Y22
INTE Y23
```

Appendix 7: The Solution to the Master Problem for the Starting Model

Global optimal solution found at step: 4
Objective value: 4731.500
Branch count: 0

Variable	Value	Reduced Cost
Z	4731.500	0.0000000
Y13	1.000000	1065.000
Y21	1.000000	2600.000
Y22	1.000000	66.00000

Appendix 8: The MIP formulation to the Starting Model

$$\begin{aligned}
 \text{MIN} \quad & -6.5 Z3 - 7.5 Z2 - 8 Z1 + 3.8 X032 + 5 X031 + 2 X030 + 3.8 X023 \\
 & + 4 X021 + 3.5 X020 + 5 X013 + 4 X012 + 3 X010 + 2 X003 \\
 & + 3.5 X002 + 3 X001 + 3.8 X232 + 5 X231 + 2 X230 + 3.8 X223 \\
 & + 4 X221 + 3.5 X220 + 5 X213 + 4 X212 + 3 X210 + 2 X203 \\
 & + 3.5 X202 + 3 X201 + 3.8 X132 + 5 X131 + 2 X130 + 3.8 X123 \\
 & + 4 X121 + 3.5 X120 + 5 X113 + 4 X112 + 3 X110 + 2 X103 \\
 & + 3.5 X102 + 3 X101
 \end{aligned}$$

SUBJECT TO

- 2) $X230 + X220 + X210 + X130 + X120 + X110 = 2$
- 3) $X031 + X021 + X001 + X231 + X221 + X201 + X131 + X121 + X101 = 1$
- 4) $X032 + X012 + X002 + X232 + X212 + X202 + X132 + X112 + X102 = 1$
- 5) $X023 + X013 + X003 + X223 + X213 + X203 + X123 + X113 + X103 = 1$
- 6) $X203 + X202 + X201 + X103 + X102 + X101 = 2$
- 7) $X013 + X012 + X010 + X213 + X212 + X210 + X113 + X112 + X110 = 1$
- 8) $X023 + X021 + X020 + X223 + X221 + X220 + X123 + X121 + X120 = 1$
- 9) $X032 + X031 + X030 + X232 + X231 + X230 + X132 + X131 + X130 = 1$
- 10) $X131 + X121 - X113 - X112 - X110 + X101 = 0$
- 11) $X231 + X221 - X213 - X212 - X210 + X201 = 0$
- 12) $X031 + X021 - X013 - X012 - X010 + X001 = 0$
- 13) $X132 - X123 - X121 - X120 + X112 + X102 = 0$
- 14) $X232 - X223 - X221 - X220 + X212 + X202 = 0$
- 15) $X032 - X023 - X021 - X020 + X012 + X002 = 0$
- 16) $X132 - X131 - X130 + X123 + X113 + X103 = 0$
- 17) $X232 - X231 - X230 + X223 + X213 + X203 = 0$
- 18) $X032 - X031 - X030 + X023 + X013 + X003 = 0$
- 19) $X113 + X112 + X110 \geq 0$
- 20) $X213 + X212 + X210 \geq 0$
- 21) $X013 + X012 + X010 \geq 1$
- 22) $X123 + X121 + X120 \geq 0$
- 23) $X223 + X221 + X220 \geq 1$
- 24) $X023 + X021 + X020 \geq 0$
- 25) $X132 + X131 + X130 \geq 1$
- 26) $X232 + X231 + X230 \geq 0$
- 27) $X032 + X031 + X030 \geq 0$
- 28) $X132 + X131 + X123 + X121 + X113 + X112 \leq 2$
- 29) $X232 + X231 + X223 + X221 + X213 + X212 \leq 2$
- 30) $X032 + X031 + X023 + X021 + X013 + X012 \leq 2$

31) $Z_3 + Z_2 + Z_1 \leq 400$
32) $Z_1 \leq 400$
33) $Z_2 \leq 50$
34) $Z_3 \leq 350$
35) $Z_1 \leq 0$
36) $Z_3 \leq 150$
37) $Z_2 \leq 200$
END

Appendix 9: The Solution to MIP for the Starting Model

Global optimal solution found at step: 15

Objective value: 4717.000

Variable	Value	Reduced Cost
X103	1.000000	0.000000
X130	1.000000	0.000000
X202	1.000000	0.000000
X220	1.000000	0.000000
X001	1.000000	0.000000
X010	1.000000	0.000000
X	1.000000	0.000000
Z2	50.00000	0.000000
Z3	150.0000	0.000000
Y01	1.000000	0.000000
Y22	1.000000	0.000000
Y13	1.000000	0.000000

And the dual variables associated with vehicle capacity constraints are as follows:

35	0.000000	8.000000
36	0.000000	6.500000

Appendix 10 : The formulation of First milestone

$$\begin{aligned} \text{MIN} \quad & 3 X_{101} + 3.5 X_{102} + 2 X_{103} + 3 X_{110} + 4 X_{112} + 5 X_{113} + 3.5 X_{120} \\ & + 4 X_{121} + 3.8 X_{123} + 2 X_{130} + 5 X_{131} + 3.8 X_{132} + 3 X_{201} \\ & + 3.5 X_{202} + 2 X_{203} + 3 X_{210} + 4 X_{212} + 5 X_{213} + 3.5 X_{220} \\ & + 4 X_{221} + 3.8 X_{223} + 2 X_{230} + 5 X_{231} + 3.8 X_{232} - 4 Z_{32} \\ & - 3.75 Z_{31} + .5 Z_{30} - 2 Z_{23} - 3.75 Z_{21} + .5 Z_{20} - 2 Z_{13} - 4 Z_{12} \\ & + .5 Z_{10} - 2 Z_{03} - 4 Z_{02} - 3.75 Z_{01} \end{aligned}$$

SUBJECT TO

- 2) $X_{101} + X_{102} + X_{103} + X_{201} + X_{202} + X_{203} = 2$
- 3) $X_{110} + X_{112} + X_{113} + X_{210} + X_{212} + X_{213} = 1$
- 4) $X_{120} + X_{121} + X_{123} + X_{220} + X_{221} + X_{223} = 1$
- 5) $X_{130} + X_{131} + X_{132} + X_{230} + X_{231} + X_{232} = 1$
- 6) $X_{110} + X_{120} + X_{130} + X_{210} + X_{220} + X_{230} = 2$
- 7) $X_{101} + X_{121} + X_{131} + X_{201} + X_{221} + X_{231} = 1$
- 8) $X_{102} + X_{112} + X_{132} + X_{202} + X_{212} + X_{232} = 1$
- 9) $X_{103} + X_{113} + X_{123} + X_{203} + X_{213} + X_{223} = 1$
- 10) $X_{101} - X_{110} - X_{112} - X_{113} + X_{121} + X_{131} = 0$
- 11) $X_{201} - X_{210} - X_{212} - X_{213} + X_{221} + X_{231} = 0$
- 12) $X_{102} + X_{112} - X_{120} - X_{121} - X_{123} + X_{132} = 0$
- 13) $X_{202} + X_{212} - X_{220} - X_{221} - X_{223} + X_{232} = 0$
- 14) $X_{103} + X_{113} + X_{123} - X_{130} - X_{131} - X_{132} = 0$
- 15) $X_{203} + X_{213} + X_{223} - X_{230} - X_{231} - X_{232} = 0$
- 16) $X_{110} + X_{112} + X_{113} - Y_{101} - Y_{121} - Y_{131} \geq 0$
- 17) $X_{210} + X_{212} + X_{213} - Y_{201} - Y_{221} - Y_{231} \geq 0$
- 18) $X_{120} + X_{121} + X_{123} - Y_{102} - Y_{112} - Y_{132} \geq 0$
- 19) $X_{220} + X_{221} + X_{223} - Y_{202} - Y_{212} - Y_{232} \geq 0$
- 20) $X_{130} + X_{131} + X_{132} - Y_{103} - Y_{113} - Y_{123} \geq 0$
- 21) $X_{230} + X_{231} + X_{232} - Y_{203} - Y_{213} - Y_{223} \geq 0$
- 22) $X_{101} + X_{102} + X_{103} - Y_{110} - Y_{120} - Y_{130} \geq 0$
- 23) $X_{201} + X_{202} + X_{203} - Y_{210} - Y_{220} - Y_{230} \geq 0$
- 24) $X_{101} + X_{102} + X_{103} \leq 1$
- 25) $X_{201} + X_{202} + X_{203} \leq 1$
- 26) $X_{110} + X_{120} + X_{130} \leq 1$
- 27) $X_{210} + X_{220} + X_{230} \leq 1$
- 28) $X_{112} + X_{121} \leq 1$
- 29) $X_{212} + X_{221} \leq 1$
- 30) $X_{113} + X_{131} \leq 1$
- 31) $X_{213} + X_{231} \leq 1$
- 32) $X_{123} + X_{132} \leq 1$
- 33) $X_{223} + X_{232} \leq 1$
- 34) $X_{112} + X_{113} + X_{121} + X_{123} + X_{131} + X_{132} \leq 2$
- 35) $X_{212} + X_{213} + X_{221} + X_{223} + X_{231} + X_{232} \leq 2$
- 36) $3.5 X_{101} + 4 X_{102} + 2.5 X_{103} + 3.5 X_{110} + 4.5 X_{112} + 5.5 X_{113}$

- $+ 4 X120 + 4.5 X121 + 4.3 X123 + 2.5 X130 + 5.5 X131 + 4.3 X132$
 ≤ 12
- 37) $3.5 X201 + 4 X202 + 2.5 X203 + 3.5 X210 + 4.5 X212 + 5.5 X213$
 $+ 4 X220 + 4.5 X221 + 4.3 X223 + 2.5 X230 + 5.5 X231 + 4.3 X232$
 ≤ 12
- 38)- $20 X101 + X101DEL + X101UP \leq 0$
 39)- $20 X102 + X102DEL + X102UP \leq 0$
 40)- $20 X103 + X103DEL + X103UP \leq 0$
 41)- $20 X110 + X110DEL + X110UP \leq 0$
 42)- $20 X112 + X112DEL + X112UP \leq 0$
 43)- $20 X113 + X113DEL + X113UP \leq 0$
 44)- $20 X120 + X120DEL + X120UP \leq 0$
 45)- $20 X121 + X121DEL + X121UP \leq 0$
 46)- $20 X123 + X123DEL + X123UP \leq 0$
 47)- $20 X130 + X130DEL + X130UP \leq 0$
 48)- $20 X131 + X131DEL + X131UP \leq 0$
 49)- $20 X132 + X132DEL + X132UP \leq 0$
 50)- $20 X201 + X201DEL + X201UP \leq 0$
 51)- $20 X202 + X202DEL + X202UP \leq 0$
 52)- $20 X203 + X203DEL + X203UP \leq 0$
 53)- $20 X210 + X210DEL + X210UP \leq 0$
 54)- $20 X212 + X212DEL + X212UP \leq 0$
 55)- $20 X213 + X213DEL + X213UP \leq 0$
 56)- $20 X220 + X220DEL + X220UP \leq 0$
 57)- $20 X221 + X221DEL + X221UP \leq 0$
 58)- $20 X223 + X223DEL + X223UP \leq 0$
 59)- $20 X230 + X230DEL + X230UP \leq 0$
 60)- $20 X231 + X231DEL + X231UP \leq 0$
 61)- $20 X232 + X232DEL + X232UP \leq 0$
 66)- $X0UP - X0DEL + Z00 = 0$
 67)- $X1UP - X1DEL + Z11 = 0$
 68)- $X2UP - X2DEL + Z22 = 0$
 69)- $X3UP - X3DEL + Z33 = 0$
 70)- $X201DEL - X201UP - X101DEL - X101UP + Z01 = 0$
 71)- $X202DEL - X202UP - X102DEL - X102UP + Z02 = 0$
 72)- $X203DEL - X203UP - X103DEL - X103UP + Z03 = 0$
 73)- $X210DEL - X210UP - X110DEL - X110UP + Z10 = 0$
 74)- $X212DEL - X212UP - X112DEL - X112UP + Z12 = 0$
 75)- $X213DEL - X213UP - X113DEL - X113UP + Z13 = 0$
 76)- $X220DEL - X220UP - X120DEL - X120UP + Z20 = 0$
 77)- $X221DEL - X221UP - X121DEL - X121UP + Z21 = 0$
 78)- $X223DEL - X223UP - X123DEL - X123UP + Z23 = 0$
 79)- $X230DEL - X230UP - X130DEL - X130UP + Z30 = 0$
 80)- $X231DEL - X231UP - X131DEL - X131UP + Z31 = 0$
 81)- $X232DEL - X232UP - X132DEL - X132UP + Z32 = 0$

- 82) $Z_{00} + Z_{03} + Z_{02} + Z_{01} \leq 17$
83) $Z_{11} + Z_{13} + Z_{12} + Z_{10} \leq 7$
84) $Z_{22} + Z_{23} + Z_{21} + Z_{20} \leq 11$
85) $Z_{33} + Z_{32} + Z_{31} + Z_{30} \leq 10$
86) $Z_{30} + Z_{20} + Z_{10} \leq 10$
87) $Z_{31} + Z_{21} + Z_{01} \leq 6$
88) $Z_{32} + Z_{12} + Z_{02} \leq 4$
89) $Z_{23} + Z_{13} + Z_{03} \leq 4$
90) $z_{10} * Y_{110} + z_{20} * Y_{120} + z_{30} * Y_{130} + z_{01} * Y_{101} + z_{21} * Y_{121} + z_{31} * Y_{131} + z_{31} * Y_{102} + z_{12} * Y_{112} + z_{32} * Y_{132} + z_{03} * Y_{103} + z_{13} * Y_{113} + z_{23} * Y_{123} \leq 7$
91) $z_{10} * Y_{210} + z_{20} * Y_{220} + z_{30} * Y_{230} + z_{01} * Y_{201} + z_{21} * Y_{221} + z_{31} * Y_{231} + z_{02} * Y_{202} + z_{12} * Y_{212} + z_{32} * Y_{232} + z_{03} * Y_{203} + z_{13} * Y_{213} + z_{23} * Y_{223} \leq 7$
92) $-50 Y_{110} - 50 Y_{210} + Z_{10} \leq 0$
93) $-50 Y_{112} - 50 Y_{212} + Z_{12} \leq 0$
94) $-50 Y_{113} - 50 Y_{213} + Z_{13} \leq 0$
95) $-50 Y_{120} - 50 Y_{220} + Z_{20} \leq 0$
96) $-50 Y_{121} - 50 Y_{221} + Z_{21} \leq 0$
97) $-50 Y_{123} - 50 Y_{223} + Z_{23} \leq 0$
98) $-50 Y_{130} - 50 Y_{230} + Z_{30} \leq 0$
99) $-50 Y_{131} - 50 Y_{231} + Z_{31} \leq 0$
100) $-50 Y_{132} - 50 Y_{232} + Z_{32} \leq 0$
101) $-50 Y_{101} - 50 Y_{201} + Z_{01} \leq 0$
102) $-50 Y_{102} - 50 Y_{202} + Z_{02} \leq 0$
103) $-50 Y_{103} - 50 Y_{203} + Z_{03} \leq 0$
104) $50 Y_1 + Z_{31} + Z_{21} + Z_{01} \geq 0$
105) $-50 Y_1 + Z_{13} + Z_{12} + Z_{10} \leq 0$
106) $50 Y_1 + Z_{31} + Z_{21} + Z_{01} \leq 50$
107) $-50 Y_1 + Z_{13} + Z_{12} + Z_{10} \geq -50$
108) $50 Y_2 + Z_{32} + Z_{12} + Z_{02} \geq 0$
109) $-50 Y_2 + Z_{23} + Z_{21} + Z_{20} \leq 0$
110) $50 Y_2 + Z_{32} + Z_{12} + Z_{02} \leq 50$
111) $-50 Y_2 + Z_{23} + Z_{21} + Z_{20} \geq -50$
112) $50 Y_3 + Z_{23} + Z_{13} + Z_{03} \geq 0$
113) $-50 Y_3 + Z_{32} + Z_{31} + Z_{30} \leq 0$
114) $50 Y_3 + Z_{23} + Z_{13} + Z_{03} \leq 50$
115) $-50 Y_3 + Z_{32} + Z_{31} + Z_{30} \geq -50$
116) $50 Y_0 + Z_{30} + Z_{20} + Z_{10} \geq 0$
117) $-50 Y_0 + Z_{03} + Z_{02} + Z_{01} \leq 0$
118) $50 Y_0 + Z_{30} + Z_{20} + Z_{10} \leq 50$
119) $-50 Y_0 + Z_{03} + Z_{02} + Z_{01} \geq -50$
120) $50 Y_1 + Z_{11} \geq 7$
121) $-50 Y_1 + Z_{11} \geq -50$
122) $50 Y_2 + Z_{22} \geq 11$
123) $-50 Y_2 + Z_{22} \geq -50$
124) $50 Y_3 + Z_{33} \geq 10$

125) $- 50 Y3 + Z33 \geq - 50$
 126) $50 Y0 + Z00 \geq 15$
 127) $- 50 Y0 + Z00 \geq - 50$
 128) $X0DEL + X203DEL + X202DEL + X201DEL + X103DEL + X102DEL$
 $+ X101DEL \leq 2$
 129) $X1DEL + X213DEL + X212DEL + X210DEL + X113DEL + X112DEL$
 $+ X110DEL \leq 1$
 130) $X2DEL + X223DEL + X221DEL + X220DEL + X123DEL + X121DEL$
 $+ X120DEL \leq 2$
 131) $X3DEL + X232DEL + X231DEL + X230DEL + X132DEL + X131DEL$
 $+ X130DEL \leq 1$
 132) $X0UP + X203UP + X202UP + X201UP + X103UP + X102UP + X101UP$
 ≤ 15
 133) $X1UP + X213UP + X212UP + X210UP + X113UP + X112UP + X110UP$
 ≤ 6
 134) $X2UP + X223UP + X221UP + X220UP + X123UP + X121UP + X120UP$
 ≤ 9
 135) $X3UP + X232UP + X231UP + X230UP + X132UP + X131UP + X130UP$
 ≤ 9
 136) $Y101 + Y121 + Y131 + Y201 + Y221 + Y231 \leq 1$
 137) $Y102 + Y112 + Y132 + Y202 + Y212 + Y232 \leq 1$
 138) $Y103 + Y113 + Y123 + Y203 + Y213 + Y223 \leq 1$
 END
 INTE X101
 INTE X102
 INTE X103
 INTE X110
 INTE X112
 INTE X113
 INTE X120
 INTE X121
 INTE X123
 INTE X130
 INTE X131
 INTE X132
 INTE X201
 INTE X202
 INTE X203
 INTE X210
 INTE X212
 INTE X213
 INTE X220
 INTE X221
 INTE X223
 INTE X230

INTE X231
INTE X232
INTE Y101
INTE Y121
INTE Y131
INTE Y201
INTE Y221
INTE Y231
INTE Y102
INTE Y112
INTE Y132
INTE Y202
INTE Y212
INTE Y232
INTE Y103
INTE Y113
INTE Y123
INTE Y203
INTE Y213
INTE Y223
INTE Y110
INTE Y120
INTE Y130
INTE Y210
INTE Y220
INTE Y230
INTE Y1
INTE Y2
INTE Y3
INTE Y0

Appendix 11: The result (1) to the First Milestone

Local optimal solution found at step: 442
 Objective value: 133.5500
 Branch count: 30

Variable	Value	Reduced Cost
X102	1.000000	0.0000000
X123	1.000000	0.0000000
X130	1.000000	0.0000000
X201	1.000000	0.0000000
X210	1.000000	0.0000000
Z01	6.000000	0.0000000
Z02	4.000000	0.0000000
X0	1.000000	0.0000000
X1	1.000000	0.0000000
X2	1.000000	0.0000000
X3	1.000000	0.0000000
Y201	1.000000	0.0000000
Y102	1.000000	0.0000000
Y103	1.000000	0.0000000
X102UP	4.000000	0.0000000
X123UP	0.1665335E-14	0.0000000
X201UP	4.000000	0.0000000
X201DEL	2.000000	0.0000000
Z00	7.000000	0.0000000
X0UP	7.000000	0.0000000
Z11	7.000000	0.0000000
X1DEL	1.000000	0.0000000
X1UP	6.000000	0.0000000
Z22	11.00000	0.0000000
X2DEL	2.000000	0.0000000
X2UP	9.000000	0.0000000
Z33	10.00000	0.0000000
X3DEL	0.9999999	0.0000000
X3UP	9.000000	0.0000000
Y3	1.000000	0.0000000
Y0	1.000000	0.0000000

Appendix 12: The result (2) to the First Milestone

Local optimal solution found at step: 402
Objective value: 133.5500
Branch count: 23

Variable	Value	Reduced Cost
X103	1.000000	0.000000
X120	1.000000	0.000000
X132	1.000000	0.000000
X201	1.000000	0.000000
X210	1.000000	0.000000
Z01	6.000000	0.000000
Z23	0.5551115E-15	0.000000
Z32	4.000000	0.000000
X0	1.000000	0.000000
X1	1.000000	0.000000
X2	1.000000	0.000000
X3	1.000000	0.000000
Y201	1.000000	0.000000
Y132	1.000000	0.000000
Y103	1.000000	0.000000
X132UP	3.000000	0.000000
X132DEL	1.000000	0.000000
X201UP	6.000000	0.000000
Z00	11.00000	0.000000
X0DEL	2.000000	0.000000
X0UP	9.000000	0.000000
Z11	7.000000	0.000000
X1DEL	1.000000	0.000000
X1UP	6.000000	0.000000
Z22	11.00000	0.000000
X2DEL	2.000000	0.000000
X2UP	9.000000	0.000000
Y3	1.000000	0.000000
Y0	1.000000	0.000000

Appendix 13: The Formulation of Second Milestone

$$\begin{aligned}
 \text{MIN} \quad & 3 X_{101} + 3.5 X_{102} + 2 X_{103} + 3 X_{110} + 4 X_{112} + 5 X_{113} + 3.5 X_{120} \\
 & + 4 X_{121} + 3.8 X_{123} + 2 X_{130} + 5 X_{131} + 3.8 X_{132} + 3 X_{201} \\
 & + 3.5 X_{202} + 2 X_{203} + 3 X_{210} + 4 X_{212} + 5 X_{213} + 3.5 X_{220} \\
 & + 4 X_{221} + 3.8 X_{223} + 2 X_{230} + 5 X_{231} + 3.8 X_{232} + 3 X_{001} \\
 & + 3.5 X_{002} + 2 X_{003} + 3 X_{010} + 4 X_{012} + 5 X_{013} + 3.5 X_{020} \\
 & + 4 X_{021} + 3.8 X_{023} + 2 X_{030} + 5 X_{031} + 3.8 X_{032} - 4 Z_{32} \\
 & - 3.75 Z_{31} + .5 Z_{30} - 2 Z_{23} - 3.75 Z_{21} + .5 Z_{20} - 2 Z_{13} - 4 Z_{12} \\
 & + .5 Z_{10} - 2 Z_{03} - 4 Z_{02} - 3.75 Z_{01}
 \end{aligned}$$

SUBJECT TO

- 2) $X_{101} + X_{102} + X_{103} + X_{201} + X_{202} + X_{203} = 2$
- 3) $X_{110} + X_{112} + X_{113} + X_{210} + X_{212} + X_{213} + X_{010} + X_{012} + X_{013} = 1$
- 4) $X_{120} + X_{121} + X_{123} + X_{220} + X_{221} + X_{223} + X_{020} + X_{021} + X_{023} = 1$
- 5) $X_{130} + X_{131} + X_{132} + X_{230} + X_{231} + X_{232} + X_{030} + X_{031} + X_{032} = 1$
- 6) $X_{110} + X_{120} + X_{130} + X_{210} + X_{220} + X_{230} = 2$
- 7) $X_{101} + X_{121} + X_{131} + X_{201} + X_{221} + X_{231} + X_{001} + X_{021} + X_{031} = 1$
- 8) $X_{102} + X_{112} + X_{132} + X_{202} + X_{212} + X_{232} + X_{002} + X_{012} + X_{032} = 1$
- 9) $X_{103} + X_{113} + X_{123} + X_{203} + X_{213} + X_{223} + X_{003} + X_{013} + X_{023} = 1$
- 10) $X_{101} - X_{110} - X_{112} - X_{113} + X_{121} + X_{131} = 0$
- 11) $X_{201} - X_{210} - X_{212} - X_{213} + X_{221} + X_{231} = 0$
- 12) $X_{001} - X_{010} - X_{012} - X_{013} + X_{021} + X_{031} = 0$
- 13) $X_{102} + X_{112} - X_{120} - X_{121} - X_{123} + X_{132} = 0$
- 14) $X_{202} + X_{212} - X_{220} - X_{221} - X_{223} + X_{232} = 0$
- 15) $X_{002} + X_{012} - X_{020} - X_{021} - X_{023} + X_{032} = 0$
- 16) $X_{103} + X_{113} + X_{123} - X_{130} - X_{131} - X_{132} = 0$
- 17) $X_{203} + X_{213} + X_{223} - X_{230} - X_{231} - X_{232} = 0$
- 18) $X_{003} + X_{013} + X_{023} - X_{030} - X_{031} - X_{032} = 0$
- 19) $X_{110} + X_{112} + X_{113} - Y_{101} - Y_{121} - Y_{131} \geq 0$
- 20) $X_{210} + X_{212} + X_{213} - Y_{201} - Y_{221} - Y_{231} \geq 0$
- 21) $X_{010} + X_{012} + X_{013} - Y_{001} - Y_{021} - Y_{031} \geq 0$
- 22) $X_{120} + X_{121} + X_{123} - Y_{102} - Y_{112} - Y_{132} \geq 0$
- 23) $X_{220} + X_{221} + X_{223} - Y_{202} - Y_{212} - Y_{232} \geq 0$
- 24) $X_{020} + X_{021} + X_{023} - Y_{002} - Y_{012} - Y_{032} \geq 0$
- 25) $X_{130} + X_{131} + X_{132} - Y_{103} - Y_{113} - Y_{123} \geq 0$
- 26) $X_{230} + X_{231} + X_{232} - Y_{203} - Y_{213} - Y_{223} \geq 0$
- 27) $X_{030} + X_{031} + X_{032} - Y_{003} - Y_{013} - Y_{023} \geq 0$
- 28) $X_{101} + X_{102} + X_{103} - Y_{110} - Y_{120} - Y_{130} \geq 0$
- 29) $X_{201} + X_{202} + X_{203} - Y_{210} - Y_{220} - Y_{230} \geq 0$
- 30) $X_{001} + X_{002} + X_{003} - Y_{010} - Y_{020} - Y_{030} \geq 0$
- 31) $X_{101} + X_{102} + X_{103} \leq 1$
- 32) $X_{201} + X_{202} + X_{203} \leq 1$
- 33) $X_{001} + X_{002} + X_{003} \leq 1$
- 34) $X_{110} + X_{120} + X_{130} \leq 1$

- 35) $X_{210} + X_{220} + X_{230} \leq 1$
36) $X_{010} + X_{020} + X_{030} \leq 1$
37) $X_{112} + X_{121} \leq 1$
38) $X_{212} + X_{221} \leq 1$
39) $X_{012} + X_{021} \leq 1$
40) $X_{113} + X_{131} \leq 1$
41) $X_{213} + X_{231} \leq 1$
42) $X_{013} + X_{031} \leq 1$
43) $X_{123} + X_{132} \leq 1$
44) $X_{223} + X_{232} \leq 1$
45) $X_{023} + X_{032} \leq 1$
46) $X_{112} + X_{113} + X_{121} + X_{123} + X_{131} + X_{132} \leq 2$
47) $X_{212} + X_{213} + X_{221} + X_{223} + X_{231} + X_{232} \leq 2$
48) $X_{012} + X_{013} + X_{021} + X_{023} + X_{031} + X_{032} \leq 2$
49) $Z_{03} + Z_{02} + Z_{01} \leq 17$
50) $Z_{13} + Z_{12} + Z_{10} \leq 7$
51) $Z_{23} + Z_{21} + Z_{20} \leq 11$
52) $Z_{32} + Z_{31} + Z_{30} \leq 10$
53) $Z_{30} + Z_{20} + Z_{10} \leq 10$
54) $Z_{31} + Z_{21} + Z_{01} \leq 6$
55) $Z_{32} + Z_{12} + Z_{02} \leq 4$
56) $Z_{23} + Z_{13} + Z_{03} \leq 4$
57) $Z_{01} Y_{001} + Z_{21} Y_{021} + Z_{31} Y_{031} + Z_{02} Y_{002} + Z_{12} Y_{012} + Z_{32} Y_{032} + Z_{03} Y_{003} + Z_{13} Y_{013} + Z_{23} Y_{023} + Z_{10} Y_{010} + Z_{20} Y_{020} + Z_{30} Y_{030} < 0$;
58) $Z_{01} Y_{101} + Z_{21} Y_{121} + Z_{31} Y_{131} + Z_{02} Y_{102} + Z_{12} Y_{112} + Z_{32} Y_{132} + Z_{03} Y_{103} + Z_{13} Y_{113} + Z_{23} Y_{123} + Z_{10} Y_{110} + Z_{20} Y_{120} + Z_{30} Y_{130} \leq 7$
59) $Z_{01} Y_{201} + Z_{21} Y_{221} + Z_{31} Y_{231} + Z_{02} Y_{202} + Z_{12} Y_{212} + Z_{32} Y_{232} + Z_{03} Y_{203} + Z_{13} Y_{213} + Z_{23} Y_{223} + Z_{10} Y_{210} + Z_{20} Y_{220} + Z_{30} Y_{230} \leq 7$
64) $-50 Y_{110} - 50 Y_{210} + Z_{10} \leq 0$
65) $-50 Y_{112} - 50 Y_{212} + Z_{12} \leq 0$
66) $-50 Y_{113} - 50 Y_{213} + Z_{13} \leq 0$
67) $-50 Y_{120} - 50 Y_{220} + Z_{20} \leq 0$
68) $-50 Y_{121} - 50 Y_{221} + Z_{21} \leq 0$
69) $-50 Y_{123} - 50 Y_{223} + Z_{23} \leq 0$
70) $-50 Y_{130} - 50 Y_{230} + Z_{30} \leq 0$
71) $-50 Y_{131} - 50 Y_{231} + Z_{31} \leq 0$
72) $-50 Y_{132} - 50 Y_{232} + Z_{32} \leq 0$
73) $-50 Y_{101} - 50 Y_{201} + Z_{01} \leq 0$
74) $-50 Y_{102} - 50 Y_{202} + Z_{02} \leq 0$
75) $-50 Y_{103} - 50 Y_{203} + Z_{03} \leq 0$
76) $Y_{101} + Y_{121} + Y_{131} + Y_{201} + Y_{221} + Y_{231} + Y_{001} + Y_{021} + Y_{031} = 1$
77) $Y_{102} + Y_{112} + Y_{132} + Y_{202} + Y_{212} + Y_{232} + Y_{002} + Y_{012} + Y_{032} = 1$
78) $Y_{103} + Y_{113} + Y_{123} + Y_{203} + Y_{213} + Y_{223} + Y_{003} + Y_{013} + Y_{023} = 1$
79) $Y_{110} + Y_{120} + Y_{130} + Y_{210} + Y_{220} + Y_{230} + Y_{010} + Y_{020} + Y_{030} = 1$
END

INTE X101
INTE X102
INTE X103
INTE X110
INTE X112
INTE X113
INTE X120
INTE X121
INTE X123
INTE X130
INTE X131
INTE X132
INTE X201
INTE X202
INTE X203
INTE X210
INTE X212
INTE X213
INTE X220
INTE X221
INTE X223
INTE X230
INTE X231
INTE X232
INTE X001
INTE X002
INTE X003
INTE X010
INTE X012
INTE X013
INTE X020
INTE X021
INTE X023
INTE X030
INTE X031
INTE X032
INTE Y101
INTE Y121
INTE Y131
INTE Y201
INTE Y221
INTE Y231
INTE Y001
INTE Y021
INTE Y031

INTE Y102
INTE Y112
INTE Y132
INTE Y202
INTE Y212
INTE Y232
INTE Y002
INTE Y012
INTE Y032
INTE Y103
INTE Y113
INTE Y123
INTE Y203
INTE Y213
INTE Y223
INTE Y003
INTE Y013
INTE Y023
INTE Y110
INTE Y120
INTE Y130
INTE Y210
INTE Y220
INTE Y230
INTE Y010
INTE Y020
INTE Y030

Appendix 14: The Solution to the Second milestone (without generalized Benders'

Decomposition)

Local optimal solution found at step: 3625

Objective value: 127.5500

Branch count: 304

Variable	Value	Reduced Cost
X101	1.000000	0.0000000
X110	1.000000	0.0000000
X202	1.000000	0.0000000
X223	1.000000	0.0000000
X230	1.000000	0.0000000
Z01	6.000000	0.0000000
Z12	4.000000	0.0000000
Z13	3.000000	0.0000000
X0	1.000000	0.0000000
X1	1.000000	0.0000000
X2	1.000000	0.0000000
X3	1.000000	0.0000000
Y101	1.000000	0.0000000
Y212	1.000000	0.0000000
Y213	1.000000	0.0000000
Y230	1.000000	0.0000000

Appendix 15: The formulation of TSP1 for Second milestone

$$\begin{aligned} \text{MIN} \quad & 3.8 X_{032} + 5 X_{031} + 2 X_{030} + 3.8 X_{023} + 4 X_{021} + 3.5 X_{020} + 5 X_{013} \\ & + 4 X_{012} + 3 X_{010} + 2 X_{003} + 3.5 X_{002} + 3 X_{001} + 3.8 X_{232} \\ & + 5 X_{231} + 2 X_{230} + 3.8 X_{223} + 4 X_{221} + 3.5 X_{220} + 5 X_{213} \\ & + 4 X_{212} + 3 X_{210} + 2 X_{203} + 3.5 X_{202} + 3 X_{201} + 3.8 X_{132} \\ & + 5 X_{131} + 2 X_{130} + 3.8 X_{123} + 4 X_{121} + 3.5 X_{120} + 5 X_{113} \\ & + 4 X_{112} + 3 X_{110} + 2 X_{103} + 3.5 X_{102} + 3 X_{101} \end{aligned}$$

SUBJECT TO

- 2) $X_{203} + X_{202} + X_{201} + X_{103} + X_{102} + X_{101} = 2$
- 3) $X_{013} + X_{012} + X_{010} + X_{213} + X_{212} + X_{210} + X_{113} + X_{112} + X_{110} = 1$
- 4) $X_{023} + X_{021} + X_{020} + X_{223} + X_{221} + X_{220} + X_{123} + X_{121} + X_{120} = 1$
- 5) $X_{032} + X_{031} + X_{030} + X_{232} + X_{231} + X_{230} + X_{132} + X_{131} + X_{130} = 1$
- 6) $X_{230} + X_{220} + X_{210} + X_{130} + X_{120} + X_{110} = 2$
- 7) $X_{031} + X_{021} + X_{001} + X_{231} + X_{221} + X_{201} + X_{131} + X_{121} + X_{101} = 1$
- 8) $X_{032} + X_{012} + X_{002} + X_{232} + X_{212} + X_{202} + X_{132} + X_{112} + X_{102} = 1$
- 9) $X_{023} + X_{013} + X_{003} + X_{223} + X_{213} + X_{203} + X_{123} + X_{113} + X_{103} = 1$
- 10) $X_{131} + X_{121} - X_{113} - X_{112} - X_{110} + X_{101} = 0$
- 11) $X_{231} + X_{221} - X_{213} - X_{212} - X_{210} + X_{201} = 0$
- 12) $X_{031} + X_{021} - X_{013} - X_{012} - X_{010} + X_{001} = 0$
- 13) $X_{132} - X_{123} - X_{121} - X_{120} + X_{112} + X_{102} = 0$
- 14) $X_{232} - X_{223} - X_{221} - X_{220} + X_{212} + X_{202} = 0$
- 15) $X_{032} - X_{023} - X_{021} - X_{020} + X_{012} + X_{002} = 0$
- 16) $X_{132} - X_{131} - X_{130} + X_{123} + X_{113} + X_{103} = 0$
- 17) $X_{232} - X_{231} - X_{230} + X_{223} + X_{213} + X_{203} = 0$
- 18) $X_{032} - X_{031} - X_{030} + X_{023} + X_{013} + X_{003} = 0$
- 19) $X_{113} + X_{112} + X_{110} \geq 0$
- 20) $X_{213} + X_{212} + X_{210} \geq 0$
- 21) $X_{013} + X_{012} + X_{010} \geq 1$
- 22) $X_{123} + X_{121} + X_{120} \geq 0$
- 23) $X_{223} + X_{221} + X_{220} \geq 1$
- 24) $X_{023} + X_{021} + X_{020} \geq 0$
- 25) $X_{132} + X_{131} + X_{130} \geq 1$
- 26) $X_{232} + X_{231} + X_{230} \geq 0$
- 27) $X_{032} + X_{031} + X_{030} \geq 0$
- 28) $X_{103} + X_{102} + X_{101} \geq 1$
- 29) $X_{203} + X_{202} + X_{201} \geq 0$
- 30) $X_{003} + X_{002} + X_{001} \geq 0$
- 31) $X_{103} + X_{102} + X_{101} \leq 1$
- 32) $X_{203} + X_{202} + X_{201} \leq 1$
- 33) $X_{003} + X_{002} + X_{001} \leq 1$
- 34) $X_{130} + X_{120} + X_{110} \leq 1$
- 35) $X_{230} + X_{220} + X_{210} \leq 1$
- 36) $X_{003} + X_{002} + X_{001} \leq 1$

- 37) $X_{121} + X_{112} \leq 1$
 - 38) $X_{221} + X_{212} \leq 1$
 - 39) $X_{021} + X_{012} \leq 1$
 - 40) $X_{131} + X_{113} \leq 1$
 - 41) $X_{231} + X_{213} \leq 1$
 - 42) $X_{031} + X_{013} \leq 1$
 - 43) $X_{132} + X_{123} \leq 1$
 - 44) $X_{232} + X_{223} \leq 1$
 - 45) $X_{032} + X_{023} \leq 1$
 - 46) $X_{132} + X_{131} + X_{123} + X_{121} + X_{113} + X_{112} \leq 2$
 - 47) $X_{232} + X_{231} + X_{223} + X_{221} + X_{213} + X_{212} \leq 2$
 - 48) $X_{032} + X_{031} + X_{023} + X_{021} + X_{013} + X_{012} \leq 2$
- END

Appendix 16: The solution to the TSP 1 for Second milestone

Global optimal solution found at step: 12
Objective value: 17.00000

Variable	Value	Reduced Cost
X103	1.000000	0.000000
X130	1.000000	0.000000
X202	1.000000	0.000000
X220	1.000000	0.000000
X001	1.000000	0.000000
X010	1.000000	0.000000
Y001	1.000000	0.000000
Y202	1.000000	0.000000
Y113	1.000000	0.000000
Y130	1.000000	0.000000

Appendix 17: The formulation of Inventory/Allocation for Second Milestone

MODEL :

MIN =

$$\begin{aligned} & - 3.75*z_{01} - 4*z_{02} - 2*z_{03} + 0.5*z_{10} - 4*z_{12} - 2*z_{13} \\ & + 0.5*z_{20} - 3.75*z_{21} - 2*z_{23} + 0.5*z_{30} - 3.75*z_{31} - 4*z_{32} \\ & + 48.75*x_0 + 18*x_1 + 51*x_2 + 39*x_3 ; \end{aligned}$$

$$z_{01} + z_{02} + z_{03} \leq 17;$$

$$z_{10} + z_{12} + z_{13} \leq 7;$$

$$z_{20} + z_{21} + z_{23} \leq 11;$$

$$z_{30} + z_{31} + z_{32} \leq 10;$$

$$z_{10} + z_{20} + z_{30} \leq 10;$$

$$z_{01} + z_{21} + z_{31} \leq 6;$$

$$z_{02} + z_{12} + z_{32} \leq 4;$$

$$z_{03} + z_{13} + z_{23} \leq 4;$$

$$\begin{aligned} & z_{10}*y_{010} + z_{20}*y_{020} + z_{30}*y_{030} + \\ & z_{01}*y_{001} + z_{21}*y_{021} + z_{31}*y_{031} + \\ & z_{02}*y_{002} + z_{12}*y_{012} + z_{32}*y_{032} + \\ & z_{03}*y_{003} + z_{13}*y_{013} + z_{23}*y_{023} < 0; \end{aligned}$$

$$\begin{aligned} & z_{10}*y_{110} + z_{20}*y_{120} + z_{30}*y_{130} + \\ & z_{01}*y_{101} + z_{21}*y_{121} + z_{31}*y_{131} + \\ & z_{02}*y_{102} + z_{12}*y_{112} + z_{32}*y_{132} + \\ & z_{03}*y_{103} + z_{13}*y_{113} + z_{23}*y_{123} < 7; \end{aligned}$$

$$\begin{aligned} & z_{10}*y_{210} + z_{20}*y_{220} + z_{30}*y_{230} + \\ & z_{01}*y_{201} + z_{21}*y_{221} + z_{31}*y_{231} + \\ & z_{02}*y_{202} + z_{12}*y_{212} + z_{32}*y_{232} + \\ & z_{03}*y_{203} + z_{13}*y_{213} + z_{23}*y_{223} < 7; \end{aligned}$$

$$z_{10} - 50*y_{110} - 50*y_{210} < 0;$$

$$z_{12} - 50*y_{112} - 50*y_{212} < 0;$$

$$z_{13} - 50*y_{113} - 50*y_{213} < 0;$$

$$z_{20} - 50*y_{120} - 50*y_{220} < 0;$$

$$z_{21} - 50*y_{121} - 50*y_{221} < 0;$$

$$z_{23} - 50*y_{123} - 50*y_{223} < 0;$$

$$z_{30} - 50*y_{130} - 50*y_{230} < 0;$$

$$z_{31} - 50*y_{131} - 50*y_{231} < 0;$$

$$z_{32} - 50*y_{132} - 50*y_{232} < 0;$$

$z_{01} - 50*y_{101} - 50*y_{201} < 0;$
 $z_{02} - 50*y_{102} - 50*y_{202} < 0;$
 $z_{03} - 50*y_{103} - 50*y_{203} < 0;$

$x_0 = 1;$
 $x_1 = 1;$
 $x_2 = 1;$
 $x_3 = 1;$

$y_{101} = 0;$
 $y_{121} = 0;$
 $y_{131} = 0;$
 $y_{201} = 0;$
 $y_{221} = 0;$
 $y_{231} = 0;$
 $y_{001} = 1;$
 $y_{021} = 0;$
 $y_{031} = 0;$

$y_{102} = 0;$
 $y_{112} = 0;$
 $y_{132} = 0;$
 $y_{202} = 1;$
 $y_{212} = 0;$
 $y_{232} = 0;$
 $y_{002} = 0;$
 $y_{012} = 0;$
 $y_{032} = 0;$

$y_{103} = 0;$
 $y_{113} = 1;$
 $y_{123} = 0;$
 $y_{203} = 0;$
 $y_{213} = 0;$
 $y_{223} = 0;$
 $y_{003} = 0;$
 $y_{013} = 0;$
 $y_{023} = 0;$

$y_{110} = 0;$
 $y_{120} = 0;$
 $y_{130} = 1;$
 $y_{210} = 0;$
 $y_{220} = 0;$

y230 = 0;
y010 = 0;
y020 = 0;
y030 = 0;

Appendix 18: The solution to the Inventory/Allocation for Second Milestone

Global optimal solution found at step: 6
 Objective value: 132.7500

Variable	Value	Reduced Cost
Z02	4.000000	0.0000000
Z13	4.000000	0.0000000
X0	1.000000	0.0000000
X1	1.000000	0.0000000
X2	1.000000	0.0000000
X3	1.000000	0.0000000
Y001	1.000000	0.0000000
Y130	1.000000	0.0000000
Y113	1.000000	0.0000000
Y202	1.000000	0.0000000

Row	Slack or Surplus	Dual Price
8	0.0000000	4.000000
9	0.0000000	2.000000
10	0.0000000	3.750000
13	0.0000000	0.0000000

Appendix19: The formulation of Master Problem for Second Milestone

MIN Z

SUBJECT TO

- 2)- 100.5 Y001 - 100.5 Y021 - 100.5 Y031 - 101 Y002 - 101 Y012
 - 101 Y032 - 92 Y003 - 92 Y013 - 92 Y023 - 48.75 Y101
 - 48.75 Y121 - 48.75 Y131 - 35 Y102 - 35 Y112 - 35 Y132 - 33 Y103
 - 33 Y113 - 33 Y123 - 48.75 Y201 - 48.75 Y221 - 48.75 Y231
 - 38.75 Y202 - 38.75 Y212 - 38.75 Y232 - 31 Y203 - 31 Y213
 - 31 Y223 + Z >= 25.5
- 3)- 101 Y002 - 101 Y012 - 101 Y032 - 92 Y003 - 92 Y013 - 92 Y023
 - 35 Y102 - 35 Y112 - 35 Y132 - 33 Y103 - 33 Y113 - 33 Y123
 - 38.5 Y202 - 38.5 Y212 - 38.5 Y232 - 31 Y203 - 31 Y213 - 31 Y223
 - 39 Y010 - 39 Y020 - 39 Y030 - 18 Y110 - 18 Y120 - 18 Y130
 - 18 Y210 - 18 Y220 - 18 Y230 + Z >= 25.5
- 4)- 97.5 Y001 - 97.5 Y021 - 97.5 Y031 - 92 Y003 - 92 Y013 - 92 Y023
 - 48.75 Y101 - 48.75 Y121 - 48.75 Y131 - 33 Y103 - 33 Y113
 - 33 Y123 - 52.25 Y201 - 52.25 Y221 - 52.25 Y231 - 31 Y203
 - 31 Y213 - 31 Y223 - 39 Y010 - 39 Y020 - 39 Y030 - 18 Y110
 - 18 Y120 - 18 Y130 - 18 Y210 - 18 Y220 - 18 Y230 + Z >= 25.5
- 5)- 97.5 Y001 - 97.5 Y021 - 97.5 Y031 - 102 Y002 - 102 Y012 - 102 Y032
 - 48.75 Y101 - 48.75 Y121 - 48.75 Y131 - 53 Y102 - 53 Y112
 - 53 Y132 - 52.25 Y201 - 52.25 Y221 - 52.25 Y231 - 51 Y202
 - 51 Y212 - 51 Y232 - 39 Y010 - 39 Y020 - 39 Y030 - 18 Y110
 - 18 Y120 - 18 Y130 - 18 Y210 - 18 Y220 - 18 Y230 + Z >= 25.5
- 6) 4 Y002 + 4 Y013 <= 0
- 7) 4 Y102 + 4 Y113 <= 7
- 8) 4 Y202 + 4 Y213 <= 7
- 9) Y001 + Y021 + Y031 + Y101 + Y121 + Y131 + Y201 + Y221 + Y231 = 1
- 10) Y002 + Y012 + Y032 + Y102 + Y112 + Y132 + Y202 + Y212 + Y232 = 1
- 11) Y003 + Y013 + Y023 + Y103 + Y113 + Y123 + Y203 + Y213 + Y223 = 1
- 12) Y010 + Y020 + Y110 + Y120 + 2 Y130 + Y210 + Y220 + Y230 = 1
- 13) Y001 + Y021 + Y031 + Y002 + Y012 + Y032 + Y003 + Y013 + Y023
 + Y010 + Y020 + Y130 <= 2
- 14) Y101 + Y121 + Y131 + Y102 + Y112 + Y132 + Y103 + Y113 + Y123
 + Y120 + Y130 + Y210 <= 2
- 15) Y201 + Y221 + Y231 + Y202 + Y212 + Y232 + Y203 + Y213 + Y223
 + Y210 + Y220 + Y230 <= 2

END

INTE Y001

INTE Y021

INTE Y031

INTE Y002

INTE Y012

INTE Y032
INTE Y003
INTE Y013
INTE Y023
INTE Y101
INTE Y121
INTE Y131
INTE Y102
INTE Y112
INTE Y132
INTE Y103
INTE Y113
INTE Y123
INTE Y201
INTE Y221
INTE Y231
INTE Y202
INTE Y212
INTE Y232
INTE Y203
INTE Y213
INTE Y223
INTE Y010
INTE Y020
INTE Y030
INTE Y110
INTE Y120
INTE Y130
INTE Y210
INTE Y220
INTE Y230

Appendix 20: The solution to the Master Problem for Second Milestone

Global optimal solution found at step: 40

Objective value: 144.0000

Branch count: 3

Variable	Value	Reduced Cost
Z	144.0000	0.0000000
Y121	1.000000	48.75000
Y232	1.000000	38.75000
Y223	1.000000	31.00000
Y110	1.000000	0.0000000

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